CS2123 Data Structures
Graphs

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Tasks of the Week

- Introduce undirected and directed graphs
- Learn basic graph algorithms
- Review for the final

Graph Concepts

Mathematically, a graph $G$ has a set of vertexes $V(G) = \{1, 2, 3, \ldots, N\}$ and a set of edges $E(G) = \{(u, v) | u, v \in V(G)\}$

- Undirected Graph: edges have no direction $(u, v) \Leftrightarrow (v, u)$
- Directed Graph: edges have directions $(u, v)$ does not imply $(v, u)$
Basic Concepts
Graph Concepts

Example Graphs

Vertexes: \( V(G) = \{1, 2, 3, 4, 5, 6\} \)
Edges: \( E(G) = \{(1, 4), (1, 6), (2, 3), (2, 4), (3, 4), (3, 5), (4, 6)\} \)

Terminal

Degree: \# of edges of a node
- Ex: \( \text{deg}(1) = 2, \text{deg}(5) = 1, \text{deg}(4) = 4 \)
- In-degree, out-degree

Adjacent nodes: linked by an edge
- Ex: 1 and 4, 2 and 3

Incident
- Ex: 2 is incident of edge (2, 3)

Graph Properties

Example
In an undirected graph,

\[
|E(G)| \leq \frac{|V(G)| \times (|V(G)| - 1)}{2} \sum_{v \in V(G)} \text{deg}(v) = 2 \times |E(G)|
\]

There are many such properties. For details, read books on Graph Theory.

Representations

- Adjacency Matrix

Example
Undirected Graph

\[
A = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]
Basic Concepts

Graph Concepts

Representations

- Adjacency Matrix

**Example**

Directed Graph

\[
A = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

- Adjacency Lists

**Example**

Undirected Graph

Graph Algorithms

- Graph Traversals: visit every vertex once
  - Breadth-first
  - Depth-first

- Connectivity
  - A graph is connected if and only if the breadth-first traversal visits \(|V(G)|\) vertexes

- Shortest paths
  - Assume each edge has a weight. Find a minimum weight path between
    - a single pair of vertices
    - between all pairs of vertices

Week15: Graphs

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