#13. A group of $n$, $n>2$, people play "odd person out" to decide who buys drinks for the group. Each person flips a fair coin simultaneously, and the person that has an outcome different from the rest of them buys drinks. If there is no single odd person out, then the coins are flipped again. This is repeated until an odd person out is identified.

Let $X$ be the number of times the coin-flip experiment is conducted.

$$P(X = 1) = \frac{n}{2^n}$$

Each round is a Bernoulli Trial

$$p = \text{prob. of success} = \frac{n}{2^n}$$

$$(b) \quad P(X = k) = q^{k-1} p = \left(1 - \frac{n}{2^n}\right)^{k-1} \cdot \frac{n}{2^{n-1}}$$

$X \sim \text{Geometric} \left(\frac{n}{2^n}\right)$

(c) $\mathbb{E}(X)$
The mean grade in a test was 72.

Standard deviation = 9

Top 10% students receive an A's.

What is the min. score a student must get to secure an A.

\[ \mu = 72 \quad \sigma = 9 \]

\( X \sim \text{Student score} \)

\[ Z = \frac{X - \mu}{\sigma} = \frac{X - 72}{9} \]

is the corresponding standardized RV

\[ P(Z > a) = 0.1 \quad \text{find 'a'} \]

\[ a = \text{Table}^{-1}(0.4) = 1.28 \]

\[ X > \mu + a \sigma = 72 + (1.28)(9) = 72 + 10.9 = 82.9 \]
Find the area under the curve between $Z = -1.20$ and $Z = 2.40$

\[
P(-1.2 \leq Z \leq 2.4) = P(-1.2 \leq Z \leq 0) + P(0 \leq Z \leq 2.4)
\]

\[
= P(0 \leq Z \leq 1.2) \quad \text{(due to symmetry)}
+ P(0 \leq Z \leq 2.4)
\]

\[
= \text{Table entry}(1.2) + \text{Table}(2.4)
= 0.7849 + 0.4918 = 0.8767
\]

Test mean score 78, $\mu = 10$

Determine the std. score of a student's score of 95

\[
X = \text{score of a student} \quad X \sim N(78, 10^2)
\]

\[
Z = \text{std. score} = \frac{X - \mu}{\sigma} = \frac{X - 78}{10}
\]

\[
X = 95, \quad Z = \frac{95 - 78}{10} = 1.7
\]

(b) Std. score of -0.5 - what is the actual score?

\[
Z = -0.5 \Rightarrow X = \mu + Z\sigma = 78 + (-0.5)10 = 73
\]
Quick Probability Estimates

Let $X$ be a nonnegative RV with finite mean $\mu$.

**Markov Inequality**

$$P(X \geq k) \leq \frac{\mu}{k}, \quad k > 0$$

Let $X$ be a RV with finite mean $\mu$ and variance $\sigma^2$.

**Chebyshev Inequality**

$$P(|X - \mu| \geq k) \leq \frac{\sigma^2}{k^2}, \quad k > 0$$

$\sigma^2 = \text{V}(X)$

[KR] Section 7.4, Problem 35

Use Chebyshev's inequality to find an upper bound on the prob. that the # of tails in $n$ coin flips deviates from the mean by more than $5\sqrt{n}$.

$$p = \frac{1}{2} \quad n \quad X = \# \text{ of tails in } n \text{ flips}$$

$$\mu = E(X) = np = \frac{n}{2} \quad X \sim \text{Bin}(n, \frac{1}{2})$$

$$\text{V}(X) = npq = n \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{n}{4}$$

$$P(|X - \mu| \geq 5\sqrt{n}) \leq \frac{\text{V}(X)}{(5\sqrt{n})^2} = \frac{n/4}{25n} = \frac{1}{100} < 0.01$$
The number of soda cans filled in a day is a random variable with an average of 10,000 cans with a variance of 1,000 cans^2.

a. Estimate the probability that more than 11,000 cans are filled in a day.

\[ X = \text{# of cans filled in a day} \]

\[ \mu = 10,000 = E(X) \quad \sigma^2 = 1,000 \]

\[ P( X > 11,000 ) \leq \frac{\mu}{11,000} = \frac{10,000}{11,000} = \frac{10}{11}. \]

b'. Estimate the probability that more than 11000 cans or fewer than 9000 cans are filled in a day.

\[ X - \mu < -1000 \quad -1000 < X - \mu < 1000 \quad X - \mu > 1000 \]

\[ P \left( \left| X - \mu \right| > 1000 \right) \leq \frac{\sigma^2}{1000^2} = \frac{1000}{1000^2} = 0.001 \]

b. What is the probability that the number of cans produced in a day is between 9,000 and 11,000?

\[ P \left( (X - \mu) < 1000 \right) = 1 - P \left( (X - \mu) > 1000 \right) \]

\[ > 1 - 0.001 = 0.999 \]
Law of Large Numbers and Central Limit Theorem

Let \( S_n = x_1 + x_2 + \cdots + x_n \), where each \( x_i \) has finite mean \( \mu \) and variance \( \sigma^2 \).

\( x_i \) are independent and identically distributed.

\[
\lim_{n \to \infty} \left( \frac{S_n - n\mu}{\sqrt{n} \sigma} \right) = 0
\]

Law of large numbers

\[
\left( \lim_{n \to \infty} \frac{S_n}{n} \right) \to \mu
\]

\[
\frac{S_n - n\mu}{\sqrt{n} \sigma}
\]

is asymptotically a standard normal RV for sufficiently large \( n \).

Central Limit Theorem.