Joint Rate and Power Control with Pricing

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Abstract— **Next generation wireless systems will be required to support heterogeneous services with different transmission rates that include real time multimedia transmissions, as well as non-real time data transmissions. In order to provide flexible transmission rates to each terminal, efficient use of system resources requires transmission rate control in addition to power control. In this paper, we present an algorithm for joint transmission rate and power control based on a non-cooperative game theoretic approach. A new utility function that includes pricing is defined for joint transmission rate and power control, and a detailed analysis of the existence and uniqueness of Nash equilibrium for the non-cooperative joint transmission rate and power control game with pricing is presented. Numerical results obtained from simulations that compare the proposed algorithm with alternative algorithms on joint rate and power control are also presented in the paper.**

I. Introduction

The increasing demand for data services in current and future generation wireless communication systems has generated the need for more efficient use of radio resources. We note that emerging data services and Internet access in cellular phones has resulted in new algorithms for power control for information sources other than the voice services, since transmitting speech signals through wireless links is different from transmitting wireless data [5]. In wireless data services the principal purpose of power control is to provide each signal with adequate quality without causing unnecessary interference to other signals. In addition, efficient power control helps to minimize the battery drain in mobile and portable terminals. Recent results on the power control algorithms for wireless data can be found in [1]–[3], [5], [7], [9].

Next generation wireless systems will include heterogeneous services such as real time multimedia transmissions, as well as non-real time data transmissions. In order to provide flexible transmission rates to each terminal, efficient use of system resources in this case requires transmission rate control in addition to power control, and this has resulted in the need for joint rate and power control in wireless systems. We note that very little work has been done in this field, and related work can be found in [4], [8]. In [4], joint rate and power control is approched from a game-theoretic perspective, and modeled as two different games. One disadvantage of the algorithm in [4] is that all terminals must first find the rate of transmission and then apply power control to allocate the powers. An alternative algorithm for joint rate and power control based on game theory is the one proposed in [8]. We note that this does not consider the concepts of pricing, and in this case the terminal closer to the base station achieves higher rates at lower powers, while terminals farther away from the base station transmit at full power with very low rates. We also note that when no pricing is included, each terminal maximizes its corresponding utility by adjusting rate and power, without taking into account the potential harmful effect (amount of interference) it has on other users in the system.

An effective way to deal with harmful effects in non-cooperative environments is to introduce *pricing* to control the overall satisfaction experienced by users in the system, as shown by their corresponding utility functions. In this paper, we introduce a new utility function for joint rate and power control in wireless systems. We formulate the problem as a non-cooperative game in terms of intrinsic properties of the channel (SIR and power), and we decouple it from the lower layer decisions such as modulation and coding. We introduce also the concept of pricing and require the users to maximize net utilities, that is utility minus pricing. We show that there exists a Nash equilibrium in the non-cooperative joint rate and power control game with pricing, and we also show by simulation that this equilibrium is superior to that of the joint rate and power control game without pricing. We propose a joint rate and power control algorithm which finds an optimal rate of transmission and allocates the power required to transmit by solving a single game theory problem.

The paper is organized as follows: in Section 2 we describe the system model and introduce the new utility function. In Section 3 we present the formulation of joint rate and power control as a non-cooperative game. In Section 4 we introduce pricing, and investigate existence and uniqueness of Nash equilibrium in the non-cooperative game with pricing. Simulation setup and numerical results are presented Section 5, followed by final conclusions and future work in Section 6.

II. System Model and Utility Function

We consider a single-cell of a CDMA wireless communication system with N mobile terminals (users) transmitting data to the same base station, in which the SIR corresponding to a given user j is expressed as [5]

$$\gamma_j = \frac{W}{r_j} \frac{h_j p_j}{\sum_{k=1, j\neq k}^{N} h_k p_k + \sigma^2} \qquad j = 1, 2, ..., N \quad (1)$$

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where W is the available spread-spectrum bandwidth (in [Hz]), σ^2 is the Additive White Gaussian Noise (AWGN) power spectral density at the receiver, h_j is the path gain of user j to the base station, and r_j, respectively p_j, are the transmission rate, respectively power, of user j. This expression assumes that users in the CDMA system are assigned pseudorandom noise (PN) sequences, and that conventional matched filter detectors are used at the receiver [5].

The objective of each user is to optimize its transmission rate and power in a distributed manner, such that its corresponding utility is maximized. The concept of utility functions originates in microeconomics, and refers to the level of satisfaction a decision maker receives as a result of its actions. Different utility functions have been considered for wireless communication systems [3], [5]. The utility function considered in [5] depends on the modulation technique, coding, and packet size. The utility function considered in [3] does not include the lower layer characterstics such as modulation technique and coding. In general, utility functions used in power control must satisfy two main properties [5]: 1) For fixed transmit powers, the utility increases with the increase in the SIR of the terminal, and 2) For fixed SIRs, the utility decreases as the transmitted power increases. In addition, for rate control the utility function must also satisfy: a) For fixed transmission rates, the utility increases with the increase in the SIR of the terminal, and b) For fixed SIRs, the utility increases as the rate increases. All these observations have prompted us to consider the following utility function in our game theoretic approach to joint rate and power control

$$u_j = \frac{r_j \ln(K\gamma_j)}{p_j} \text{[bits/sec/Watt]} \tag{2}$$

where K is a constant that determines the quality of service requirements. The utility function u_j of user j, can be regarded as the ratio of throughput, T_j, to the transmitted power, p_j, with $T_j = r_j f(\gamma_j)$ and $f(\gamma_j)$ the frame success rate of user j. Here we consider the frame success rate as a logarithmic function of the SIR $f(\gamma_j) = \ln(K\gamma_j)$, which implies that the value of K is

$$K = \frac{e^{f(\gamma_j)}}{\gamma_j} \tag{3}$$

Therefore the general expression of the considered utility function is

$$u = \frac{r \ln(K\gamma)}{p} \text{[bits/Joule]} \tag{4}$$

We note that the utility is not defined when transmit power $p_j = 0$, and we impose a lower bound $\hat{p}$ on all user transmit powers, that is $p_j \geq \hat{p}$, which is the minimum power level for transmission for all users in the system.

III. Formulation of a Non-Cooperative Game

Let $G = [\mathcal{N}, \{P_j, R_j\}, \{u_j(\cdot)\}]$ denote the *non-cooperative rate and power control game* (NRPG), where $\mathcal{N} = \{1, 2, ..., N\}$ is the index set for the active mobile users in the cell, P_j is the strategy set of user powers, R_j is the strategy set of user rates, and $u_j(\cdot)$ is the utility function. Each user selects a rate $r_j \in R_j$ and a power $p_j \in P_j$. Let the rate vector $\mathbf{r} = (r_1, r_2, ..., r_N)^T \in R_j^N$ ($R_j^N = R_j \times R_j \times ... \times R_j$), power vector $\mathbf{p} = (p_1, p_2, ..., p_N)^T \in P_j^N$ ($P_j^N = P_j \times P_j \times ... \times P_j$) (where T represents the transpose operator) denote the outcome of the game in terms of selected rate and power levels of all the users. The resulting utility of user j obtained by expending p_j is given in equation (2). Here we assume that the strategy spaces R_j and P_j of each user are compact and convex sets with maximum and minimum constraints. For user j we consider strategy spaces $R_j = [\underline{r_j}, \overline{r_j}]$ and $P_j = [\underline{p_j}, \overline{p_j}]$ which are closed intervals, the smallest power $\underline{p_j} \geq \hat{p}$.

In a distributed rate and power control game, each user adjusts rate r_j and power p_j in order to maximize the utility u_j. Formally, the NRPG is expressed as

$$\max_{r_j \in R_j, p_j \in P_j} u_j(\mathbf{r}, \mathbf{p}), \text{for all} j \in N \tag{5}$$

where u_j is given in (2) and $R_j = [\underline{r_j}, \overline{r_j}]$ and $P_j = [\underline{p_j}, \overline{p_j}]$ are the strategy spaces of user j. The optimization problem for user j is to find a transmission rate $\tilde{r}_j$ from the strategy space R_j, that maximizes the utility function u_j. The maximum rate occurs at a point for which the partial derivative of u_j with respect to r_j is zero ($\partial u_j / \partial r_j = 0$). We obtain the condition for maximizing the transmission rate

$$\ln(K\gamma_j) - 1 = 0 \tag{6}$$

For power allocation, the optimization problem for user j is to find the power level $\tilde{p}_j$ from the strategy space P_j, that maximizes the utility function u_j ($\partial u_j / \partial p_j = 0$), which leads to the same condition for maximizing utility as in equation (6). Thus, we note that maximum utility occurs under the same condition for both rate and power control.

The condition for optimum rate and power control can be used to obtain the value of the constant K. By rearranging equation (6), the value of K is obtained

$$K = \frac{e}{\tilde{\gamma}} \tag{7}$$

where $\tilde{\gamma}$ is the target SIR, which can be assumed to be transmitted to each user by the base station. We note that the value of K in (7) is obtained when there are no errors in the transmission. However, this is not guaranteed in real systems, and a more practical way of finding the value of K is to equate $\ln(K\gamma)$ to the probability of correct reception (P_c)

$$\ln(K\tilde{\gamma}) = P_c \quad \Longrightarrow \quad K = \frac{e^{P_c}}{\tilde{\gamma}} \tag{8}$$

From equation (8), we note that the value of the target SIR $\tilde{\gamma}$ can be determined by the value of K in

$$\tilde{\gamma} = \frac{e^{P_c}}{K} \tag{9}$$

The advantage of this procedure is that the target SIR can be determined by adjusting the value of K, a feature which is not present in previous related work [3], [5], [8]. Numerical simulations performed show that by changing the value of K

users can attain a different equilibrium SIR. We note that the base station can compute the value of K (depending on the number of active mobile users in the cell, probability of correct reception, all the practical considerations) and transmits it to the mobile users. Thus, the mobile users will employ the joint rate and power control game to achieve the QoS requirements.

The Nash equilibrium is the solution for non-cooperative game theoretic problems. A Nash equilibrium in transmission rates exists and is unique in the game $G = [\mathcal{N}, \{P_j, R_j\}, \{u_j(\cdot)\}]$, for all $j = 1, 2, ..., N$. A Nash equilibrium in transmit powers for the same game G also exists and is unique. We omit the proofs due to space restrictions.

IV. Non-Cooperative Rate and Power Control Game with Pricing

In non-cooperative rate and power control, each terminal maximizes its own utility by adjusting its rate and power, but it ignores the harm (amount of interference) it imposes on the other users. Pricing is an effective tool used to deal with these harmful effects. An efficient pricing mechanism makes the decentralized decisions compatible with the overall system efficiency by encouraging efficient sharing of the resources as opposed to the aggressive competition of the pure non-cooperative games.

We define a non-cooperative game with pricing in which the price is proportional to the rate of the terminal. Let $G_p = [\mathcal{N}, \{R_j, P_j\}, \{u_j^p(\cdot)\}]$ denote the *non-cooperative rate and power control game with rate pricing* (NRPGP), where $\mathcal{N} = \{1, 2..., N\}$ is the index set for the active mobile users in the cell, R_j is the rate strategy set, and $u_j^p(\cdot)$ is the utility function. The utilities for the NRPGP are

$$u_j^p(\mathbf{r}, \mathbf{p}) = u_j(\mathbf{r}, \mathbf{p}) - c_j(r_j) \tag{10}$$

where $c_j : R \rightarrow \Re_+$ is the pricing function of terminal $j \in \mathcal{N}$. We impose a linear pricing scheme in which the price increases monotonically with the rate of user j.

$$c_j(r_j) = cr_j$$

The pricing factor c should be tuned such that each user's self interest leads to overall improvement of the system. The NRPGP with linear pricing is considered as:

$$\text{NRPGP} \max_{r_j \in R_j, p_j \in P_j} u_j^p(\mathbf{r}, \mathbf{p}) = u_j(\mathbf{r}, \mathbf{p}) - cr_j, \forall j \in \mathcal{N} \tag{11}$$

To derive an algorithm for the NRPGP game we adopt a rate and power control algorithm in which each terminal maximizes its net utility ($u_j^p(\mathbf{r}, \mathbf{p}) = u_j^p$). This can be achieved at a point for which the partial derivative of u_j^p with respect to r_j is equal to zero.

$$\frac{\partial u_j^p}{\partial r_j} = \frac{1}{p_j}[-1 + \ln(K\gamma_j)] - c = 0 \tag{12}$$

Rearranging (12), the condition for maximizing utility with rate pricing becomes

$$\ln(K\gamma_j) - 1 - cp_j = 0 \tag{13}$$

For power allocation, the optimization problem for user j is to find the power level $\tilde{p}_j$ from the strategy space P_j, that maximizes the utility function u_j ($\frac{\partial u_j^p}{\partial p_j} = 0$). We have

$$\frac{\partial u_j^p}{\partial p_j} = r_j \left[\frac{p_j \frac{1}{K\gamma_j} K\gamma_j' - \ln(K\gamma_j)}{p_j^2} \right] \tag{14}$$

By rearranging (14), the condition for maximizing the utility becomes $\ln(K\gamma_j) - 1 = 0$, which is same as (6).

Definition 1: A rate vector $\mathbf{r} = (r_1, ..., r_N)$ is a Nash equilibrium of the NRPGP $G_p = [\mathcal{N}, \{P_j, R_j\}, \{u_j^p\}]$ if, for every $j \in \mathcal{N}$, $u_j(r_j, r_{-j}) \geq u_j(r_j', r_{-j})$ for all $r_j' \in R_j$.

Theorem 1 (Existence): A Nash equilibrium in the transmission rates exists in the game $G_p = [\mathcal{N}, \{P_j, R_j\}, \{u_j^p(\cdot)\}]$ if, for all $j = 1, 2..., N$:

1) R_j is a nonempty, convex, and compact subset of some Euclidean space $\Re^N$.
2) $u_j(\mathbf{r})$ is continuous in $\mathbf{r}$ and quasi-concave in r_j.

Proof: We assumed that each user has a strategy space that is defined by the maximum and minimum rates, and all the rates in between. So the first condition on the strategy space P_j is satisfied.

To show that a function is quasi-concave it is sufficient to show that it is concave, and we use the second derivative test for this.

$$\begin{aligned} \frac{\partial u_j^p}{\partial r_j} &= \frac{1}{p_j}\left[r_j \frac{1}{K\gamma_j} K\gamma_j' + \ln(K\gamma_j)\right] - c \\ &= \frac{1}{p_j}[-1 + \ln(K\gamma_j)] - c \end{aligned} \tag{15}$$

$$\frac{\partial^2 u_j^p}{\partial r_j^2} = \frac{1}{p_j} \frac{\partial \gamma_j / \partial r_j}{\gamma_j} = -\frac{1}{p_j r_j} \tag{16}$$

From (16), note that $\partial^2 u_j^p / \partial r_j^2 < 0$, $\forall j \in \mathcal{N}$, which means that u_j is a concave function of r_j. Therefore, this guarantees the existence of a Nash equilibrium. □

Next we prove the uniqueness of the Nash equilibrium point of the game G_p by proposing the best response of user j in game G_p similar to proposition 1.

Proposition 1: For a game $G_p = [\mathcal{N}, \{R_j, P_j\}, \{u_j^p\}]$, the best response of user j, given the transmission rates vector of other users $\mathbf{r}_j$ is given by: $\rho_j(\mathbf{r}_j) = \min(\tilde{r}_j, \overline{r}), \forall j \in N$, where $\overline{r}$ is the maximum rate of transmission of user j's strategy space R_j.

Proof: Let $\rho_j(\mathbf{r}_{-j})$ be the best response function of the j^{th} user as a best strategy that the user j can take to attain the maximum utility given the other users' strategy $\mathbf{r}_{-j}$. Formally, terminal j's best response $\rho_j : R_{-j} \rightarrow R_j$ is the mapping that assigns to each $\mathbf{r}_{-j} \in R_{-j}$ the set

$$\rho_j(\mathbf{r}_{-j}) = \{r_j \in R_j : u_j(r_j, r_{-j}) \geq u_j(r_j', r_{-j}), \forall r_j' \in R_j\} \tag{17}$$

where this is a set containing only one point. Therefore $\tilde{r}_j$ is the unconstrained maximum of the utility function u_j.

$$\tilde{r}_j = \arg \max_{r_j \in \Re^+} u_j^p \tag{18}$$

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In addition, as seen from (16), $\partial^2 u_j^p/\partial r_j^2 < 0$, $\forall r_j \in \Re^+$, which implies that the maximum is unique.

When $\tilde{r}_j$ is not feasible, each user j will transmit at the maximum rate $\overline{r}$ since the target function is increasing on the set $\{r_j : r_j < \tilde{r}_j\}$. This implies that $r_j = \overline{r}$ is the best response of user j. $\square$

Theorem 2 (Uniqueness): The Nash equilibrium of the joint rate and power control game with pricing is unique.

Proof: By Theorem 1, we know that there exists an equilibrium. Let $\tilde{\mathbf{r}}$ denote the Nash equilibrium. By definition, the Nash equilibrium has to satisfy $\tilde{\mathbf{r}} = \rho(\mathbf{r})$ where $\rho(\mathbf{r}) = (\rho_1(\mathbf{r}), \rho_2(\mathbf{r}), ...\rho_n(\mathbf{r}))$ is the best response vector of all users. The key aspect of uniqueness is to show that the best response function $\rho(\mathbf{r})$ is standard. A function is said to be *standard* if it satisfies the following properties:

- **positivity**: $\rho(\mathbf{r}) > 0$
- **monotonicity**: if $\mathbf{r} \geq \mathbf{r}'$ then $\rho(\mathbf{r}) \geq \rho(\mathbf{r}')$
- **scalability**: for all $\mu > 1$, $\mu\rho(\mathbf{r}) > \rho(\mu\mathbf{r})$

These properties can be easily verified for $\rho(\mathbf{r})$. It is shown in [6] that the fixed point $\tilde{\mathbf{r}} = \rho(\mathbf{r})$ is unique for a standard function. Therefore, the Nash equilibrium is unique. $\square$

Definition 2: A power vector $\mathbf{p} = (p_1, ..., p_n)$ is a Nash equilibrium of the NRPGP $G_p = [\mathcal{N}, \{P_j, R_j\}, \{u_j^p(\cdot)\}]$ if, for every $j \in N$, $u_j(p_j, p_{-j}) \geq u_j(p_j', p_{-j})$ for all $p_j' \in P_j$.

Theorem 3: A Nash equilibrium in the transmit powers exists in the game $G_p = [\mathcal{N}, \{P_j, R_j\}, \{u_j^p(\cdot)\}]$ if, for all $j = 1, 2, ..., N$:

1) P_j is a nonempty, convex, and compact subset of some Euclidean space $\Re^N$.
2) $u_j(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_j.

We assumed that each user has a strategy space that is defined by the maximum and minimum powers, and all the power values in between. As a consequence, the first condition on the strategy space P_j is satisfied.

$$\frac{\partial^2 u_j^p}{\partial p_j^2} = r_j \frac{\ln(K\gamma_j) - 1}{p_j^3} \tag{19}$$

We know that the probability of correct reception is always less than or equal to 1 ($P_c \leq 1 \Rightarrow log(K\gamma) \leq 1$). By using this condition in the above equation (19), we conclude that $\partial^2 u_j^p/\partial p_j^2 \leq 0$, which implies that u_j is a quasi-concave function of p_j optimized on the convex set P_j. This proves condition 2, and guarantees existence of a Nash equilibrium. $\square$

Proposition 2: For a game $G_p = [\mathcal{N}, \{P_j, R_j\}, \{u_j^p\}]$, the best response of user j, given the power vector of other users $\mathbf{p}_{-j}$ is given by: $\nu_j(\mathbf{p}_{-j}) = \min(\tilde{p}_j, \overline{p}), \forall j \in \mathcal{N}$, where $\overline{p}$ is the maximum transmit power of user j's strategy space P_j, and $\tilde{p}_j$ is the unconstrained maximum of the utility function u_j.

$$\tilde{p}_j = \arg\max_{p_j \in \Re^+} u_j^p \tag{20}$$

In addition, $\partial^2 u_j^p/\partial p_j^2 < 0$, $\forall p_j \in \Re^+$, which implies that this maximum is unique. This can be proved in a similar manner shown in proposition 1 and Theorem 2 since the best response vector $\nu(\mathbf{p}) = (\nu_1(\mathbf{p}), \nu_2(\mathbf{p}), ..., \nu_2(\mathbf{p}))$ is also a standard function. Therefore, there exists a unique Nash equilibrium point. $\square$

We now present an asynchronous rate and power control algorithm which converges to the unique Nash equilibrium point $(\tilde{\mathbf{r}}, \tilde{\mathbf{p}})$ of game G_p. Assume that the user j updates its transmission rates and powers at time instances in the set $T_j = \{t_{j1}, t_{j2} ....\}$, with $t_{jk} < t_{jk+1}$, $t_{j0} = 0$. Let ϵ be a small number (e.g. 10^{-7}). Generate a sequence of rates and powers as follows

Algorithm: Consider the non-cooperative rate and power control game (NRPGP) as given in (11). Generate a sequence of rate and powers as follows

1) Set the initial power vector at time $t = 0$ equal to any random vector p.
2) For all $j \in N$, such that $t_k \in T_j$
   a) Compute $\tilde{r}_j = \arg\max_{r_j \in R_j} u_j^p(\mathbf{r}, \mathbf{p})$. Then set the transmission rate: $r_j(t_k) = \min(\tilde{r}_j, \overline{r})$.
   b) Given the prior power $p(t_{k-1})$, compute $\tilde{p}_j = \arg\max_{p_j \in P_j} u_j^p(\mathbf{r}, \mathbf{p})$. Then set the transmit power: $p_j(t_k) = \min(\tilde{p}_j, \overline{p})$.
3) If $\|\mathbf{p}(t_k), \mathbf{p}(t_{k-1})\| \leq \epsilon$, STOP and declare the Nash equilibrium as $p(t_k)$. Else, $k = k + 1$ and go to step 2.

V. Simulation Setup and Numerical Results

We demonstrate the improvement in the performance obtained as a result of the NRPGP algorithm on a single cell CDMA system. We compare the results with those obtained using the NRPG mentioned in Section 3, and with the rate and power control algorithm described in [8]. As in [8], the users are considered at distances $d = [50, 100, 150, 200, 250, 300, 350, 400, 450, 450, 500]$ m from the base station, and are assumed to be stationary. There is no forward error correction, and the propagation model is $h_j = c/d_j^4$, where d_j is the distance between user j and the base station and $c = 0.097$. The system parameters are considered as follows:

- Spread spectrum bandwidth $W = 3.84 \times 10^6$ Hz (chip rate).
- AWGN power at the receiver $\sigma^2 = 10^{-15}$ W/Hz.
- Maximum power of each user $\overline{p} = 0.2$ Watts.
- Minimum power of each user $\underline{p} = \hat{p} = 10^{-4}$ Watts, .
- Maximum rate of transmission for each user $\overline{r} = 96$ kbps.
- Minimum rate of transmission for each user $\underline{r} = 0$ kbps.

We considered the target SIR as 12.42 (equal to the equilibrium SIR obtained by the algorithm in [5]) and then calculate the value of the constant 'K' using equation (7). The value of $K = 0.21886$. We considered the coefficient of pricing to be a small value ($c = 5$).

The equilibrium powers, rates, and SIRs for all the users are presented in Figure 1. We note that the users closer the base station transmits with higher rates and lower powers than the users farther from the base station. We also note that, while the equilibrium transmission rates and powers are the same as in NRPG and in the algorithm in [8], there is a

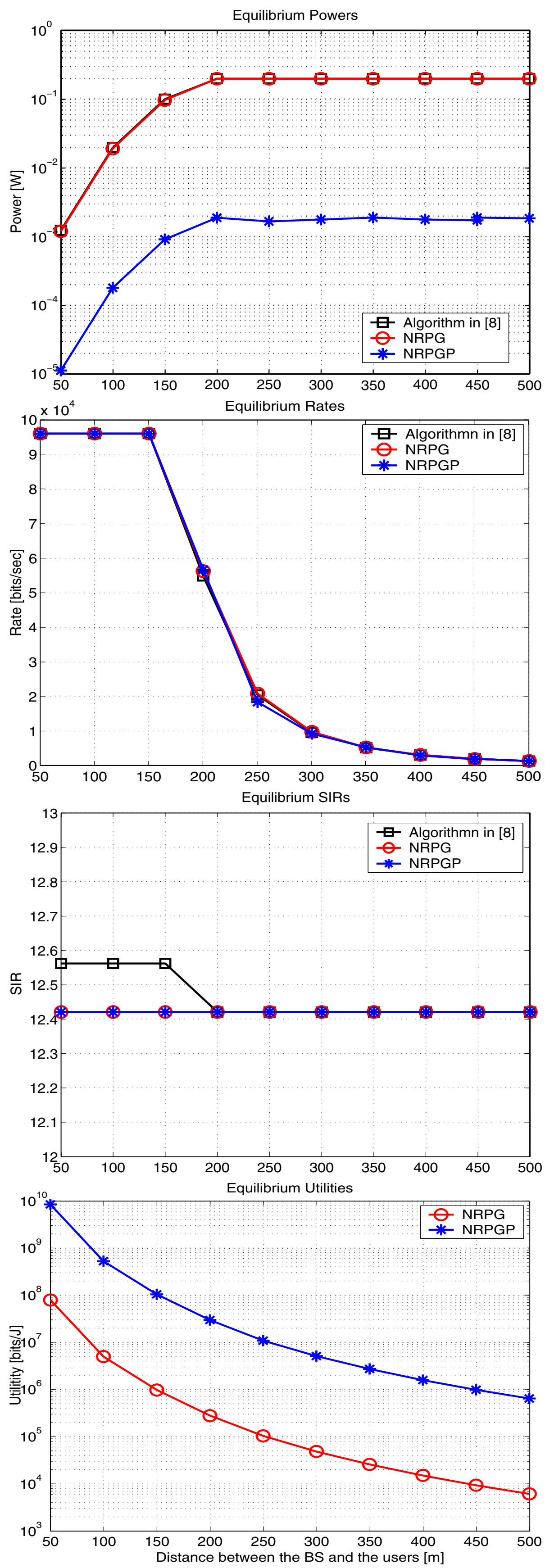

Fig. 1. Numerical results obtained from simulations.

considerable decrease in the powers of all the users under NRPGP while the transmission rates remain the same. In addition the total satisfaction experienced by user (as seen in their corresponding utility functions) increases significantly when using the NRPGP when compared with the NRPG as seen in the utility plot in Figure 1.

VI. Conclusion

In this paper we applied game theoretic concepts to model the problem of joint transmission rate and power control. A new utility function, defined as the ratio of throughput to the transmit power was developed. The maximization of the utility function results in optimal transmission rate and power. We introduced the concept of pricing the users according to their transmission rates. A distributed joint rate and power control algorithm was developed, which was based on users maximizing their own net utilities which is the difference of the utility function and the pricing function, to calculate its transmission rate and power. The non-cooperative joint rate and power control algorithm with pricing was shown to give the same equilibrium rates and SIRs as the algorithm in [8], but with a significant decrease in the powers of each user. It was also shown that there is a significant increase in the utilities of all users because of pricing.

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