any # of 1s followed by any # of 0s

\[
\begin{array}{c}
\text{in} \\
\text{111} \\
\text{100} \\
\text{110000} \\
\text{2}
\end{array}
\quad
\begin{array}{c}
\text{out} \\
\text{01...} \\
\text{01} \\
\text{110010}
\end{array}
\]

\[
A \rightarrow 1 \quad A \rightarrow 1A
\]

Derivation

\[
\begin{align*}
S & \rightarrow AB \\
A & \rightarrow 1A \\
B & \rightarrow 0B \\
S & \rightarrow AB \rightarrow 1AB \rightarrow 11AB \rightarrow 11B \rightarrow 110B \rightarrow 110
\end{align*}
\]
5. Let $G = (V, T, S, P)$ be the phrase-structure grammar with $V = \{0, 1, A, B, S\}$, $T = \{0, 1\}$, and set of productions $P$ consisting of $S \rightarrow 0A$, $S \rightarrow 1A$, $A \rightarrow 0B$, $B \rightarrow 1A$, $B \rightarrow 1$.

a) Show that 10101 belongs to the language generated by $G$.

b) Show that 10110 does not belong to the language generated by $G$.

c) What is the language generated by $G$?

\[ S \rightarrow 1A \rightarrow 10B \rightarrow 101A \rightarrow 1010B \rightarrow 10101 \]
\[ \Rightarrow 0A \Rightarrow \ldots \ldots \ldots \ldots \ldots \ldots \Rightarrow 01101 \]

starts with a 0 or a 1
followed by one or more 01s
\[(0+1) \ 01 \ (01)^*\]
13. Find a phrase-structure grammar for each of these languages.

a) the set consisting of the bit strings 0, 1, and 11
b) the set of bit strings containing only 1s
c) the set of bit strings that start with 0 and end with 1

d) the set of bit strings that consist of a 0 followed by an even number of 1s

\[ S \rightarrow 0 \quad S \rightarrow 1 \quad S \rightarrow 11 \]

\[ S \rightarrow 1 \quad S \rightarrow 1S \]

\[ S \rightarrow ABC \]
\[ A \rightarrow 0 \]
\[ C \rightarrow 1 \]
\[ B \rightarrow 1B \]
\[ B \rightarrow 0B \]
\[ B \rightarrow 1 \]

\[ S \rightarrow OBE \]
\[ B \rightarrow 1B \]
\[ B \rightarrow 0B \]
\[ B \rightarrow 1 \]

\[
\begin{array}{c|c|c}
\text{in} & \text{out} & S \\
0 & 10 & 01 \text{ 11} \\
01 & 11 & 01 \text{ 11} \\
011 & 111 & 0111 \text{ 1111} \\
0111 & 1111 & 01111 \text{ 11111} \\
\end{array}
\]

\[ S \rightarrow OA \]
\[ A \rightarrow 1A \]
\[ A \rightarrow 1A \]
\( f(5_0, 0) = s_0 \)
\( f(5_0, 1) = s_1 \)
\( f(s_1, 0) = s_0 \)
\( f(s_1, 1) = s_1 \)

\[ F = \{ s, 3 \} \]
15. Find a phrase-structure grammar for each of these languages.

a) the set of all bit strings containing an even number of 0s and no 1s
b) the set of all bit strings made up of a 1 followed by an odd number of 0s
c) the set of all bit strings containing an even number of 0s and an even number of 1s
d) the set of all strings containing 10 or more 0s and no 1s
e) the set of all strings containing more 0s than 1s
f) the set of all strings containing an equal number of 0s and 1s
g) the set of all strings containing an unequal number of 0s and 1s

[Diagrams for a) and b) are shown]
25. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain the string 101.
RES

0 ~ 003
1 ~ 113

001 ~ 30,13

0* ~ \{2,0,00,000,\ldots\}
1* ~ \{2,1,\psi,111,\ldots\}

(001)* ~ all bit strings

00 ~ 3003
01 ~ 3013

0(00)* ~ \{2,00,0000,000000,\ldots\}
0(00)* ~ \{0,000,00000,\ldots\}

(00(1(01*051)))*

10001101

128

84

141
3. Determine whether 0101 belongs to each of these regular sets.

a) 01*0*  
   0, 01, 00, 010, 011, 000  
   0101 does not belong

b) 0(11)*(01)*  
   0, 011, 001  
   no to 0101

c) (01)*(11)*  
   0, 01, 010, 011  
   yes to 0101

d) 0*10(0 ∪ 1)  
   100, 101, 0100, 0101  
   yes

e) 0*(10 ∪ 11)*  
   0* include 2 empty string

f) 01(01 ∪ 0)*  

h) 01(01 ∪ 1)*  

5. Express each of these sets using a regular expression.
   a) the set consisting of the strings 0, 1, 1, and 010
   b) the set of strings of three 0s followed by two or more 0s
   c) the set of strings of odd length
   d) the set of strings that contain exactly one 1
   e) the set of strings ending in 1 and not containing 000

a) \( 0 \cup 1 \cup 0 \cup 010 \)

b) \( 000000^* \) wrong, generates 0000 0 000000^*

c) \( (0 \cup 1) (00)^*(01) (10)^* (11)^* \)
   \( \text{can generate 01100} \)
   \( (0 \cup 1) (00 \cup 01 \cup 10 \cup 11)^* \)
   \( (0 \cup 1) (00 \cup 11)^* \)

d) \( 0^*10^* \)
5. Express each of these sets using a regular expression.
   a) the set consisting of the strings 0, 11, and 010
   b) the set of strings of three 0s followed by two or more 0s
   c) the set of strings of odd length
   d) the set of strings that contain exactly one 1
   e) the set of strings ending in 1 and not containing 000

   e) 1 1 1 1 1 1 1
      \[1001* \star001* \]
      \((10010001)^* (10010001)\)
1. Let $T$ be the Turing machine defined by the five-tuples: $(s_0, 0, s_1, 1, R)$, $(s_0, 1, s_1, 0, R)$, $(s_0, B, s_1, 0, R)$, $(s_1, 0, s_2, 1, L)$, $(s_1, 1, s_1, 0, R)$, and $(s_1, B, s_2, 0, L)$. For each of these initial tapes, determine the final tape when $T$ halts, assuming that $T$ begins in initial position.

a) \[ \cdots B B 0 \bar{1} 1 B B \cdots \]

b) \[ \cdots B B \bar{x} 0 \bar{1} B B B B \cdots \]

c) \[ \cdots B B \bar{x} \bar{x} \bar{x} 0 \bar{1} B \cdots \]

d) \[ \cdots B B B B B B B B B \cdots \]
9. Construct a Turing machine with tape symbols 0, 1, and B that, given a bit string as input, replaces all but the leftmost 1 on the tape with 0s and does not change any of the other symbols on the tape.

\[
\begin{align*}
0101 & \rightarrow 0100 \\
10111 & \rightarrow 10000 \\
00011 & \rightarrow 00010
\end{align*}
\]

So-right
hits a 1 go to 51
otherwise do nothing else

\[
\begin{align*}
S_0, 0 & \rightarrow S_0, 0, R \\
S_0, 1 & \rightarrow S_1, 1, R \\
S_0, B & \rightarrow \text{stop, } B, \text{-} \\
S_1, 0 & \rightarrow S_0, 0, R \\
S_1, 1 & \rightarrow S_0, 0, R \\
S_1, B & \rightarrow \text{stop, } B
\end{align*}
\]