Algorithms

On two occasions I have been asked, “Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?” ... I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question. (Charles Babbage)

Definition
An algorithm is a finite sequence of unambiguous instructions for performing a computation (also called solving a problem). Key properties include:
- Input. An algorithm has input (parameters) from a specific set.
- Output. An algorithm has output (return values) from a specific set.
- Definiteness. The steps of an algorithm must be unambiguous.
- Correctness/Finiteness/Effectiveness/Generality. An algorithm should produce correct output from any valid input in a finite number of steps.

Pseudocode
We will use pseudocode to describe algorithms. Read Appendix 3 (assert statement on web site).

<table>
<thead>
<tr>
<th>Statement Type</th>
<th>Pattern/Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedure</td>
<td>procedure algorithm(inputs)</td>
</tr>
<tr>
<td>Assignment</td>
<td>variable := expression</td>
</tr>
<tr>
<td>Comment</td>
<td>Book uses {}. I will use //.</td>
</tr>
<tr>
<td>Conditional</td>
<td>if condition then statement</td>
</tr>
<tr>
<td>For loop</td>
<td>Avoid using for loops.</td>
</tr>
<tr>
<td>While loop</td>
<td>while condition \n statement</td>
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<tr>
<td>Procedure call</td>
<td>algorithm(inputs)</td>
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<tr>
<td>Return</td>
<td>return expression</td>
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<tr>
<td>Assert</td>
<td>assert condition</td>
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</table>

Finding the Maximum of Two Numbers
max2 outputs the maximum of two numbers.

procedure max2(x, y)
    assert x ∈ R and y ∈ R
    if x < y
        then max := y
        else max := x
    assert max = x \ max = y
    assert max ≥ x \ max ≥ y
    return max
Assert Statements
Executing the statement “assert condition” results in an error if the condition is false.

- Assert statements should describe what should be true at that point in the program.
- An assert statement at the beginning of the procedure should describe the preconditions, what must be true of the procedure’s input.
- An assert statement at the end of the procedure should describe the postconditions, what should be true of the procedure’s output.
- Other assert statement should describe the progress of the procedure in terms of its inputs and variables.

Signum Algorithm
signum outputs the sign of a number.

procedure signum(x)
assert ??? about x
result := 0
assert ??? about result
if x < 0 then result := -1
assert ??? about result and x
if x > 0 then result := 1
assert ??? about result and x
return result

Finding the Maximum
maximum outputs the maximum element in a sequence.

procedure maximum(a_1, a_2, ..., a_n)
assert n ∈ Z^+ and \{a_1, a_2, ..., a_n\} ⊆ R
max := a_1, i := 2
assert max = a_i-1
while i ≤ n
assert max is the maximum of \{a_1, ..., a_{i-1}\}
if max < a_i then max := a_i
assert max is the maximum of \{a_1, ..., a_i\}
i := i + 1
assert max is the maximum of \{a_1, a_2, ..., a_n\}
return max

Linear Search
linear_search outputs the location of an element in a sequence, or 0 if it is not in the sequence.

procedure linear_search(k, a_1, ..., a_n)
assert ??? about k and the sequence
loc := 0, i := 1
assert ??? about loc and i
while i ≤ n ∧ loc = 0
assert ???
if k = a_i then loc := i
else i := i + 1
assert ???
assert ??? about loc
return loc