1. (10 points) Section 1.5: 20

2. (20 points) Section 1.5: 28, also indicate those statements that have different results if the domain is the integers.

3. (50 pts.) Assume the domain consists of all real numbers. Assume variables are universally quantified. In each exercise below, either prove that the statement is true, or provide a counterexample. For each proof, you must clearly indicate what kind of proof you are doing, and you must clearly indicate assumptions and what needs to be shown. You must also do at least one indirect proof and at least one proof by contradiction.

All proofs must clearly state implications by using the word “implies”.

(a) \( x < 0 \land y < 0 \) implies \( x + y < xy \)
(b) \( x = 1 \lor y = 1 \) implies \( x + y > xy \)
(c) \( x + y = xy \) implies \( 1/x + 1/y = 1 \)
(d) \( x + y = xy \) implies \( x \geq 0 \lor y \leq 1 \)
   Hint: do an indirect proof or proof by contradiction.
(e) \( x + y > xy \) implies \( x < 2 \lor y < 2 \). Hint: do an indirect proof or proof by contradiction.

4. (20 pts.) Assume the domain consists of all positive real numbers. Let \( \lg x \) be the base 2 logarithm of \( x \), that is, \( \lg x = \log_2 x \). See Appendix 2 for properties of logarithms. In each case below, either prove that the statement is true, or show that it is false (such as providing a counterexample).

All proofs must clearly state implications by using the word “implies”.

(a) \( \lg(x + y) = \lg x + \lg y \)
(b) \( \lg(x + y) = \lg x + \lg y \iff x + y = xy \)