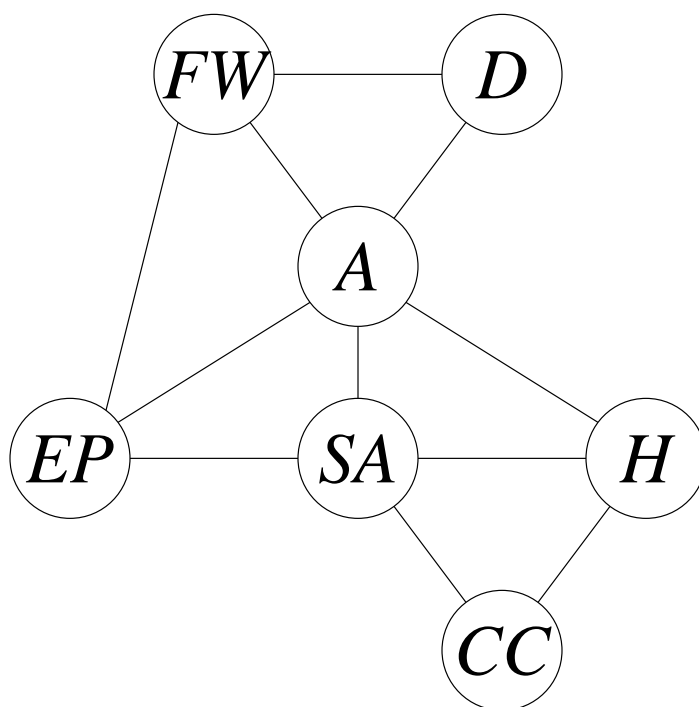


Graphs

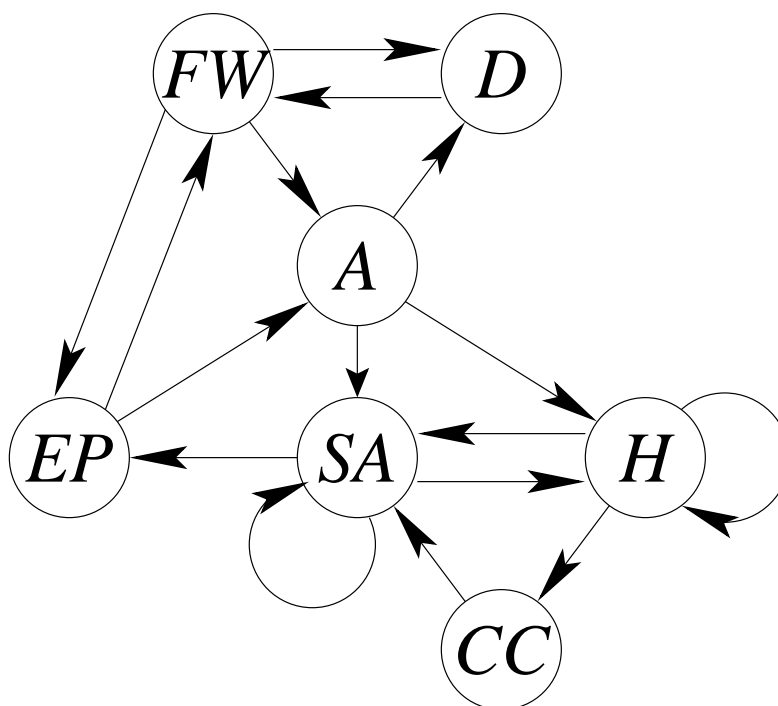
A *graph* is a set of vertices V and a set of edges E .

In a *simple graph*, each edge is a two-element subset of V .

In a *directed graph*, each edge is an ordered pair from V , i.e., the edges are a relation on V .



$$E = \{\{A, D\}, \{A, EP\}, \{A, FW\}, \{A, H\}, \dots\}$$



$$E = \{(A, D), (EP, A), (FW, A), (A, H), \dots\}$$

Some Terminology

Simple graphs:

If $\{u, v\}$ is an edge in a simple graph, then u and v are adjacent, and u and v are neighbors.

The *degree* of a vertex is how many edges it is in.

Sum of degrees equals twice the number of edges.

An even number of vertices have an odd degree.

Directed graphs:

If (u, v) is an edge in a directed graph, then u is adjacent to v , and v is adjacent from u .

The *in-degree* of a vertex is how many edges to it.

The *out-degree* of a vertex is how many edges from it.

Sum of in-degrees = sum of out-degrees = number of edges

Adjacency List Representation

Simple Graphs

Vertex	Adjacent Vertices
<i>A</i>	<i>D, EP, FW, H, SA</i>
<i>CC</i>	<i>H, SA</i>
<i>D</i>	<i>A, FW</i>
<i>EP</i>	<i>A, FW, SA</i>
<i>FW</i>	<i>A, D, EP</i>
<i>H</i>	<i>A, CC, SA</i>
<i>SA</i>	<i>A, CC, EP, H</i>

Directed Graphs, Relations

Initial Vertex	Terminal Vertices
<i>A</i>	<i>D, H, SA</i>
<i>CC</i>	<i>SA</i>
<i>D</i>	<i>FW</i>
<i>EP</i>	<i>A, FW</i>
<i>FW</i>	<i>A, D, EP</i>
<i>H</i>	<i>CC, H, SA</i>
<i>SA</i>	<i>EP, H, SA</i>

Adjacency Matrix Representation

Simple Graphs

$$\begin{array}{l}
 A \\
 CC \\
 D \\
 EP \\
 FW \\
 H \\
 SA
 \end{array}
 \left[\begin{array}{ccccccc}
 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
 1 & 1 & 0 & 1 & 0 & 1 & 0
 \end{array} \right]$$

Directed Graphs, Relations

$$\begin{array}{l}
 A \\
 CC \\
 D \\
 EP \\
 FW \\
 H \\
 SA
 \end{array}
 \left[\begin{array}{ccccccc}
 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
 0 & 0 & 0 & 1 & 0 & 1 & 1
 \end{array} \right]$$