

Homework 1

CS 3233 – Fall 2003
Tom Bylander, Instructor

assigned August 28, 2003
due September 2, 2003

1. (20 pts.) Classify each of the following compound propositions as a tautology, contradiction, or a contingency.
 - (a) $(p \wedge (p \rightarrow q)) \rightarrow q$
 - (b) $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$
 - (c) $(p \oplus q) \leftrightarrow (p \leftrightarrow q)$
 - (d) $(p \rightarrow q) \vee (q \rightarrow p)$

2. (20 pts.) Determine whether the following are true or false.
 - (a) $(\neg q \rightarrow \neg p) \equiv \neg(p \wedge \neg q)$
 - (b) $((p \rightarrow q) \oplus (q \rightarrow p)) \equiv \neg(p \leftrightarrow q)$
 - (c) $(p \rightarrow q) \wedge (q \rightarrow r) \equiv p \rightarrow r$
 - (d) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

3. (40 pts.) Let $Q(x, y)$ be the statement $x^2 > y$, and let the universe of discourse be \mathbf{Z} , the set of integers. What are the truth values for the following statements?
 - (a) $Q(1, 1)$
 - (b) $Q(0, 2)$
 - (c) $\exists x Q(x, x)$
 - (d) $\forall x Q(x, x)$
 - (e) $\forall x \exists y Q(x, y)$
 - (f) $\forall y \exists x Q(x, y)$
 - (g) $\exists x \forall y Q(x, y)$
 - (h) $\exists y \forall x Q(x, y)$

4. (20 pts.) Let the universe of discourse be \mathbf{Z} , the set of integers. What are the truth values for the following statements?
 - (a) $\forall x (x \geq -x)$
 - (b) $\forall x (x^2 + 4 \geq 4x)$
 - (c) $\exists x (x + 3 = x^2)$
 - (d) $\exists x ((x - 1)^2 = (x + 3)^2)$