

# Homework 1

CS 3233 – Fall 2003  
Tom Bylander, Instructor

assigned August 28, 2003  
due September 2, 2003

1. (20 pts.) Classify each of the following compound propositions as a tautology, contradiction, or a contingency.
  - (a)  $(p \wedge (p \rightarrow q)) \rightarrow q$
  - (b)  $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$
  - (c)  $(p \oplus q) \leftrightarrow (p \leftrightarrow q)$
  - (d)  $(p \rightarrow q) \vee (q \rightarrow p)$
  
2. (20 pts.) Determine whether the following are true or false.
  - (a)  $(\neg q \rightarrow \neg p) \equiv \neg(p \wedge \neg q)$
  - (b)  $((p \rightarrow q) \oplus (q \rightarrow p)) \equiv \neg(p \leftrightarrow q)$
  - (c)  $(p \rightarrow q) \wedge (q \rightarrow r) \equiv p \rightarrow r$
  - (d)  $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
  
3. (40 pts.) Let  $Q(x, y)$  be the statement  $x^2 > y$ , and let the universe of discourse be  $\mathbf{Z}$ , the set of integers. What are the truth values for the following statements?
  - (a)  $Q(1, 1)$
  - (b)  $Q(0, 2)$
  - (c)  $\exists x Q(x, x)$
  - (d)  $\forall x Q(x, x)$
  - (e)  $\forall x \exists y Q(x, y)$
  - (f)  $\forall y \exists x Q(x, y)$
  - (g)  $\exists x \forall y Q(x, y)$
  - (h)  $\exists y \forall x Q(x, y)$
  
4. (20 pts.) Let the universe of discourse be  $\mathbf{Z}$ , the set of integers. What are the truth values for the following statements?
  - (a)  $\forall x (x \geq -x)$
  - (b)  $\forall x (x^2 + 4 \geq 4x)$
  - (c)  $\exists x (x + 3 = x^2)$
  - (d)  $\exists x ((x - 1)^2 = (x + 3)^2)$