

Homework 3

CS 3233 – Fall 2003
Tom Bylander, Instructor

assigned September 11, 2003
due September 18, 2003

1. (25 pts.) Do Exercise 2.1.10. Write your algorithm using the book's pseudocode conventions.
2. (25 pts.) Prove that $f(n) = 4n^2 - 7n + 11$ is $O(n^2)$ by finding values for C and k and demonstrating that $f(n) \leq Cn^2$ when $n > k$.
3. (25 pts.) Prove that $f(n) = 4n^2 - 7n + 11$ is not $O(n)$ by showing that, given any C and k , you can find a value for n such that $n > k \wedge f(n) > Cn$.
4. (25 pts.) Consider the following algorithm for finding the mode in a list of integers. A mode in a list of integers is an element that occurs at least as often as any of the other elements. If there is more than one mode, this algorithm only return one of them.

```
procedure mode ( $a_1, \dots, a_n: \mathbf{Z}$ )  
  for  $i := 1$  to  $n$   
     $c_i := 0$   
   $m := 0$   
  for  $i := 1$  to  $n$   
    for  $j := 1$  to  $n$   
      if  $a_i = a_j$   
        then  $c_i := c_i + 1$   
          if  $c_i > m$  then  $m := c_i$   
  for  $i := 1$  to  $n$   
    if  $c_i = m$   
      then return  $a_i$ 
```

Answer the following questions about the algorithm.

- (a) (5 pts.) Briefly justify the correctness of the algorithm.
- (b) (10 pts.) What is the total number of assignment (worst-case)? What is the total number of comparisons (worst-case)? Do not include assignments to loop variables or comparison of loop variables in your totals.
- (c) (10 pts.) Show that the total number of assignments plus the total number of comparisons is $O(n^2)$. Note: there are $O(n)$ average-case algorithms for the mode, so this one is very inefficient.