1. (20 pts.) Do Exercise 3.4.4.

2. (40 pts.) Do Exercise 3.4.12 using mathematical induction. Clearly display the parts of your proof (predicate, basis, induction, assume, show).

3. (20 pts.) In pseudocode, provide a fast recursive algorithm for finding \( x^n \mod m \) whenever \( n, x, \) and \( m \) are positive integers. Use the fact that:

\[
x^n \mod m = \begin{cases} 
    x \mod m & \text{if } n = 1 \\
    (x^n/2 \mod m)^2 \mod m & \text{if } n \text{ is even} \\
    ((x^{n-1} \mod m) \cdot (x \mod m)) \mod m & \text{otherwise}
\end{cases}
\]

4. (20 pts.) For your algorithm in the previous exercise, write a recurrence relation for the number of mod operations that it performs.