CS 3233 Midterm II Review: Practice Problems

Chapter 3

Key Terms and Results (pp. 290–291): Term from "sequence" to $\sum_{i=1}^{n}$, from "mathematical induction" to "recursive definition of a function", "recursive algorithm", and "iteration". First two results.

Review Questions (pp. 291–292): 4, 5, 9–11, 14, 15.

Supplementary Exercises (pp. 292-297): 10–22, 28, 29, 31, 33, 41 (write a recursive algorithm), 43, 53

Chapter 4

Key Terms and Results (p. 349): All terms and results up to and including "Binomial Theorem".

Review Questions (pp. 349–350): 1-9, 12ac.

Supplementary Exercises (pp. 350–353): 1abc, 2abc, 3, 4, 6-12, 14, 15, 18, 20, 22–24, 27–29, 34–36, 39–42.

Chapter 5

Key Terms and Results (p. 394): from "probability of an event" to "random variable", "Bernoulli trial", and "probabilistic algorithm".

Review Questions (p. 395): 1–5, 8.

Supplementary Exercises (pp. 395–398): 1, 2, 11ab, 15–18, 21, 22.

Chapter 6

Key Terms and Results (p. 465): "recurrence relation" and "initial conditions for a recurrence relation".

Review Questions (p. 466): 1, 4.

Supplementary Exercises (pp. 466–468): 1-5.

An Aside

When using probabilities, one needs to be careful to make correct inferences. Suppose that 1% of some group of people have a certain disease. Suppose that a new medical test for the disease has the following properties. If a person has the disease, then the test is positive 99% of the time. If a person does not have the disease, then the test is positive 1% of the time.

This sounds like a good test, but suppose we use this test on everybody. Then 99% out of the 1% (0.99%) who have the disease test positive. Also, 1% of the 99% (also 0.99%) who do not have the disease test positive. So in reality, a random person who tests positive has a 50% probability of being healthy, i.e., the person is equally likely to be in either 0.99%. Should a costly treatment be given when 50% don't need it?

To formalize this, let E_{yes} be the event of having the disease. Let E_{no} be the event of not having the disease. Let E_{pos} be the event of a positive test. Then $p(E_{yes}|E_{pos})$ is the probability of having the disease given a positive test. We can solve the problem as follows:

$$p(E_{yes}) = 0.01 \qquad p(E_{no}) = 0.99 \qquad p(E_{pos}|E_{yes}) = 0.99 \qquad p(E_{pos}|E_{no}) = 0.01$$

$$\begin{aligned} p(E_{pos}) &= p(E_{pos} \cap E_{yes}) + p(E_{pos} \cap E_{no}) = p(E_{pos}|E_{yes})p(E_{yes}) + p(E_{pos}|E_{no})p(E_{no}) \\ &= (0.99)(0.01) + (0.01)(0.99) = 0.0198 \end{aligned}$$

$$p(E_{no}|E_{pos}) = \frac{p(E_{no} \cap E_{pos})}{p(E_{pos})} = \frac{p(E_{pos}|E_{no})p(E_{no})}{p(E_{pos})} = \frac{(0.01)(0.99)}{0.0198} = 0.5$$