Propositional Logic

A *proposition* is a statement that is T or F. Boolean variables $p, q, r, \ldots$ denote propositions. Logical operators form *compound propositions*. *Truth tables* evaluate compound propositions.

p	q	negation $\neg p$	conjunction $p \wedge q$	disjunction $p \vee q$
T	T	F	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

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p	q	exclusive or $p \oplus q$	implication $p \longrightarrow q$	biconditional $p \leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	T	F
F	F	F	T	T

Propositional Equivalences

A compound proposition is:

a *tautology* if it is always true,

a *contradiction* if it is always false, or

a *contingency*, otherwise.

p	q	$p \longrightarrow q$	$q \longrightarrow (p \longrightarrow q)$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

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p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \longrightarrow \neg p$
T	T	T	F	F
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

p	q	$p \oplus q$	$p \leftrightarrow q$	$(p \oplus q) \wedge (p \leftrightarrow q)$
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

p is *logically equivalent* to q if $p \leftrightarrow q$ is a tautology.
$p \Leftrightarrow q$ or $p \equiv q$ denotes logical equivalence.
Use truth tables to determine logical equivalence.

$p \longrightarrow q \equiv \neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$	$p \longrightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

This equivalence is one of De Morgan's laws.
The other one is $\neg(p \wedge q) \equiv \neg p \vee \neg q$