Representing Relations Using Matrices

A relation $R$ on $A = \{a_1, a_2, \ldots, a_n\}$ can be represented by a matrix $M$ by:

$$m_{ij} = \begin{cases} 
1 & \text{if } (a_i, a_j) \in R \\
0 & \text{if } (a_i, a_j) \notin R 
\end{cases}$$

E.g., $R = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$ can be represented by the matrix $M$:

$$M = \begin{bmatrix}
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 
\end{bmatrix}$$

$R^2$ can be obtained by Boolean product $M \odot M$.

Transitive Closure Algorithms

This works by repeated composite operations.

**procedure** transitive_closure

(R: $n \times n$ zero-one matrix)

{The $R$ matrix represents the relation}

{The $C$ matrix will represent the closure}

$C := R$

for $i := 2$ to $n$

$C := C \lor (C \odot R)$

return $C$
This works by “compressing paths.”

**procedure warshall**

\( (\mathbf{R}: n \times n \text{ zero-one matrix}) \)

\{The \( \mathbf{R} \) matrix represents the relation\}

\{The \( \mathbf{C} \) matrix will represent the closure\}

\( \mathbf{C} := \mathbf{R} \)

for \( k := 1 \) to \( n \)

\[ \text{for} \ i := 1 \ \text{to} \ n \]

\[ \text{for} \ j := 1 \ \text{to} \ n \]

\[ c_{i,j} := c_{i,j} \lor (c_{i,k} \land c_{k,j}) \]

return \( \mathbf{C} \)