Homework 1

CS 3343 – Spring 2016
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Your solutions must be submitted to Blackboard as a PDF document.

1. (20 pts.) Using the pseudocode conventions of the textbook, write an $\Theta(n)$ algorithm to find and return the maximum value of an array that contains $n$ numbers. Write a comment between each pair of lines describing what is true when the program reaches that point in your code.

2. (20 pts.) What is the minimum number of bits that are needed to encode an integer in the range $-1000$ to $1000$? From $-1000000$ to $1000000$? From $-n$ to $n$, where $n$ is a positive integer?

3. (20 pts.) Rank the following 12 functions by order of growth.

   \[ f_1(n) = \lg n \]
   \[ f_2(n) = n^2 \]
   \[ f_3(n) = 2^n \]

   All the functions $f_i(f_j(n))$ for $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3\}$

4. (20 pts.) Consider the insertion sort algorithm presented in the book. Assume that $A$ contains $n$ distinct numbers.

   (a) If $A$ is sorted in ascending order, determine the number of times each line is executed. Assuming that the cost of each line is $c$, what is the total running time? Show the result in a way similar to the top of p. 26.

   (b) If $A$ is sorted in descending order, determine the number of times each line is executed. Assuming that the cost of each line is $c$, what is the total running time? Show the result in a way similar to the top of p. 26.

5. (20 pts.) Suppose that you know that $f(n)$ is $O(n^2)$ and $\Omega(n)$. Give the most precise big-Oh and big-Omega for:

   (a) $100 \ast f(n)$
   (b) $f(n)/100$
   (c) $n \ast f(n)$
   (d) $f(n)/n$
   (e) $\lg(f(n))$
   (f) $f(n)/f(n)$
   (g) $(f(n))^2$
   (h) $2f(n)$
   (i) $\sum_{i=1}^{n} f(i)$
   (j) $\sum_{i=1}^{n} f(n)/i$
   (k) $\sum_{i=1}^{n} f(n)/(i^2)$