Homework 2

CS 3343 – Fall 2006 | assigned August 31, 2006
Tom Bylander, Instructor | due September 8, 2006

Your solutions must be submitted as a document to WebCT.

1. (40 pts., 2 pts. each) This is a variation of Exercise 2.1.1.

   For each of the following algorithms, indicate (i) a natural size metric for its inputs; (ii) its basic operation; (iii) whether the basic operation count can be different for inputs of the same size; (iv) an approximation of the basic operation count (worst-case) as a constant times a simple function (in or similar to those shown in Table 2.1, also in notes).

   (a) computing $n!$
   (b) finding the largest element in a list of n numbers
   (c) Euclid's algorithm
   (d) sieve of Eratothenes
   (e) pen-and-pencil algorithm for multiplying two n-digit numbers

2. (20 pts.) For each of the following functions, demonstrate how much the function's value will change if its argument is doubled.

   (a) $\log_2 n$
   (b) $\sqrt{n}$
   (c) $n \log_2 n$
   (d) n^k
   (e) 2^n

3. (20 pts.) For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ in your answers.) Prove your assertions.

   (a) $f_a(n) = (n^2 + 1)^{10}$. Hint: show $n^{20} \leq f_a(n) \leq 1024n^{20}$ when $n \geq 1$.
   (b) $f_b(n) = \sqrt{10n^2 + 7n + 3}$. Hint: show $3n \leq f_b(n) \leq 4n$ when $n \geq 7$.
   (c) $f_c(n) = 2^{n+1} + 3^{n-1}$. Hint: show $3^n/3 \leq f_c(n) \leq 3^n$ when $n \geq 5$.
   (d) $f_d(n) = \lfloor \log_2 n \rfloor$. Hint: show $(\log_2 n)/2 \leq f_d(n) \leq \log_2 n$ when $n \geq 4$.

4. (20 pts.) This is a variation of Exercise 2.3.3.

   The sample variation s^2 of n measurements $x_1, \ldots, x_n$ can be computed as:

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}$$

where $\overline{x}$ is the sample mean:

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

or s^2 can be computed as:

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 / n}{n-1}$$

(a) Provide pseudocode in the book's style for one of the two computations.

(b) As a function of n, find the number of multiplications/divisions and additions/subtractions (multiplications and divisions are usually grouped together, ditto for additions and subtractions) that are required for the computing the variance according to your pseudocode. You may ignore increments of loop variables in your counts.