Assume \( p \leq q < r \) and \( n = r - p + 1 \)

**MERGE**\((A, p, q, r)\)

1. \( n_1 = q - p + 1 \)
2. \( n_2 = r - q \)
3. let \( L[1 \ldots n_1 + 1] \) and \( R[1 \ldots n_2 + 1] \) be new arrays
4. **for** \( i = 1 \) **to** \( n_1 \)
5. \( L[i] = A[p + i - 1] \)
6. **for** \( j = 1 \) **to** \( n_2 \)
7. \( R[j] = A[q + j] \)
8. \( L[n_1 + 1] = \infty \)
9. \( R[n_2 + 1] = \infty \)
10. \( i = 1 \)
11. \( j = 1 \)
12. **for** \( k = p \) **to** \( r \)
13. **if** \( L[i] \leq R[j] \)
14. \( A[k] = L[i] \)
15. \( i = i + 1 \)
16. **else** \( A[k] = R[j] \)
17. \( j = j + 1 \)

**number of lines executed** = \( 6n + 9 \)
\textbf{\(\Theta\)-notation}\hfill \frac{3n^2 + 7n - 4}{2}

In Chapter 2, we found that the worst-case running time of insertion sort is \(T(n) = \Theta(n^2)\). Let us define what this notation means. For a given function \(g(n)\), we denote by \(\Theta(g(n))\) the \textit{set of functions}

\[\Theta(g(n)) = \{ f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0\} .\]
**Theta-notation**

In Chapter 2, we found that the worst-case running time of insertion sort is $T(n) = \Theta(n^2)$. Let us define what this notation means. For a given function $g(n)$, we denote by $\Theta(g(n))$ the set of functions

$$\Theta(g(n)) = \{ f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that}$$

$$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0 \}.$$

Claim: $5n-4$ is $\Theta(n)$ using $f(n) = 5n-4$, $g(n) = n$.

We need positive constants so that $c_1 n \leq 5n-4 \leq c_2 n$ when $n$ is “big” (not small).

Suppose $n \geq 10$ is it the case $4n \leq 5n-4 \leq 5n$

$\quad -10 \leq -n$  \quad $c_1 = 4$  \quad $c_2 = 5$

Proof: part 1: $5n-4 < 5n-4 + 4 = 5n$

part 2: $5n-4 > 5n-n$

$5n-4 > 5n-4 - 6 = 5n-10 \geq 5n-n = 4n$

So best-case running time of insertion sort is $\frac{T(n)}{n} = \Theta(n)$.
from Merge algorithm

Claim: $6n + q$ is $\Theta(n)$

Proof:

Choose $n_0 = q$, that is suppose $n \geq q$.

$6n + q \geq 6n + q - q = 6n$,

$6n \leq 6n + q = c_2n$.

Thus, $\exists c_1, c_2$ such that $c_1n \leq 6n + q \leq c_2n$ for all $n \geq q$.

So running time of merge is $\Theta(n)$.
worst-case insertion sort

\[ f(n) = \frac{3n^2}{2} + \frac{7n}{2} - 4 \text{ is } \Theta(n^2) \quad g(n) = n^2 \]

\[ n^2 \leq \left( \frac{3n^2}{2} + \frac{7n}{2} - 4 \right) \leq 2n^2 \]

suppose \( n \geq 10 \) means mean \( n^2 \geq 100 \)

\[ n^2 \geq 100 \quad \text{mean} \quad n \geq \frac{n^2}{10} \geq 10 \quad \text{mean} \]

part 1: \[ \frac{3n^2}{2} + \frac{7n}{2} - 4 < \frac{3n^2}{2} + \frac{7n}{2} \]

\[ = \frac{3n^2}{2} + \frac{7n^2}{2} = 2n^2 \]

part 2: \[ \frac{3n^2}{2} + \frac{7n}{2} - 4 \geq \frac{3n^2}{2} - 4 \geq \frac{3n^2}{2} - 50 \geq \frac{3n^2}{2} - \frac{n^2}{2} \]

\[ = n^2 \]

worst-case running sort is \( \Theta(n^2) \)