\[ f_1(n) = \log n \quad \log(n) = \log_2(n) \]
\[ f_2(n) = n^2 \]

\[ f_1(f_2(n)) = f_1(n^2) = \log(n^2) \]
\[ f_2(f_1(n)) = f_2(\log n) = (\log n)^2 \]

\[ n = 1000 \]
\[ \log(1000^2) = \log(1000000) \approx 20 \]
\[ (\log 1000)^2 \approx (10)^2 = 100 \]

\[ 2^{10} = 1024 \approx 1000 \]
\[ 2^{20} = 1024 \times 1024 = 1.048576 \approx 1,000,000 \]
\[ 2^{30} \approx 1,000,000,000 \]
\[ 2^{40} \approx 1,000,000,000,000 \]

\[ \log 1024 = 10 \quad \log 1000 \approx 10 \]
\[ \log 1,048,576 = 20 \quad \log 1,000,000 \approx 20 \]
\[ \log 1,000,000,000 \approx 30 \]
\[ \log 1,000,000,000,000 \approx 40 \]
Assume $n \geq 10$

implies

$n \geq 10$ implies $n^2 \geq 100$
$n \geq 10$ implies $n^2 \geq 10n$
$n \geq 10 \Rightarrow n \times n \geq 10 \times n \Rightarrow n^2 \geq 10n$

$n \geq 10$ implies $n^3 \geq 1000$
$n^3 \geq 100n$
$n^3 \geq 10n^2$

$n \geq 10$ implies $-n \leq -10$

Multiply both sides times a negative number reverses the inequality.

$n^2 \geq 10n$ implies $-n^2 \leq -10n$
Show \( \frac{3n^2}{2} + \frac{7n}{2} - 4 \) is \( \Theta(n^2) \)

educated guess \( n^2 \geq \frac{3n^2}{2} + \frac{7n}{2} - 4 \leq 2n^2 \)

assume \( n \geq 10 \)

to show \( \frac{3n^2}{2} + \frac{7n}{2} - 4 \leq 2n^2 \)

\[
\frac{3n^2}{2} + \frac{7n}{2} - 4 \leq \frac{3n^2}{2} + \frac{7n}{2}
\]

use \( n^2 \geq 10n \) same as \( 10n \leq n^2 \)

\[
\frac{3n^2}{2} + \frac{7n}{2} \leq \frac{3n^2}{2} + \frac{7(n^2)}{20}
\]

\[
\frac{3n^2}{2} + \frac{7(n^2)}{20} = \frac{3n^2}{2} + \frac{7n^2}{20}
\]

\[
= \left( \frac{3}{2} + \frac{7}{20} \right)n^2
\]

\[
= \frac{37}{20} n^2
\]

\[
\frac{37}{20} n^2 \leq \frac{40}{20} n^2 = 2n^2
\]
Assume \( n \geq 10 \)

To show \( \frac{3n^2}{2} + \frac{7n}{2} - 4 \geq n^2 \)

\[
\frac{3n^2}{2} + \frac{7n}{2} - 4 \geq \frac{3n^2}{2} - 4
\]

Use \( n^2 \geq 100 \) same as \( 100 \leq n^2 \)

\[
-100 \leq -n^2
\]

\[
-1 \geq -\frac{n^2}{100}
\]

\[
-4 \geq -4n^2
\]

\[
\frac{3n^2}{2} - 4 \geq \frac{3n^2}{2} - \frac{4n^2}{100}
\]

\[
\frac{3n^2}{2} - \frac{4n^2}{100} = \left( \frac{3}{2} - \frac{4}{100} \right) n^2
\]

\[
= \frac{146}{100} n^2
\]

\[
\frac{146}{100} n^2 \geq \frac{100}{100} n^2 = n^2
\]
\[ f(n) \text{ is } \Theta(g(n)) \]

\[ f(n) \text{ is similar to } g(n) \]
\[ \text{in its growth} \]
\[ \text{as } n \text{ becomes big} \]
\[ \text{being off by a factor of 2} \]
\[ \text{or 10 is not ignored} \]

\[ f(n) \text{ is } O(g(n)) \]
\[ f(n) \text{ is } \leq \text{ to } g(n) \text{ upper bound} \]
\[ \text{in its growth} \]
\[ \text{as } n \text{ becomes big} \]
\[ \text{being off by a factor of 23} \]
\[ \text{or 106 is ignored} \]

\[ f(n) \text{ is } \Omega(g(n)) \]
\[ f(n) \text{ is } \geq \text{ to } g(n) \text{ lower bound} \]
\[ \text{in its growth} \]

Insertion sort (inefficient)
\[ \text{is } O(n^2) \text{ and } \Omega(n) \]
\[ \text{not very useful (but true)} \]

Insertion sort
\[ \text{is } O(n^2) \text{ and } \Omega(n \log n) \]