Asymptotic Notation

Big Oh
Big Omega
Big Theta
little oh
little omega
Notation

Graph Illustration

Big Oh

Big Omega

Big Theta

little oh

little omega

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Graph Illustration

Big Oh

O-notation

The Θ-notation asymptotically bounds a function from above and below. When we have only an asymptotic upper bound, we use O-notation. For a given function \( g(n) \), we denote by \( O(g(n)) \) (pronounced “big oh of \( g \) of \( n \)”) or sometimes just “oh of \( g \) of \( n \)” the set of functions

\[
O(g(n)) = \{ f(n) : \text{ there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0 \}.
\]
**Big Omega**

Ω-notation

Just as \(O\)-notation provides an asymptotic upper bound on a function, \(\Omega\)-notation provides an asymptotic lower bound. For a given function \(g(n)\), we denote by \(\Omega(g(n))\) (pronounced “big-omega of \(g\) of \(n\)” or sometimes just “omega of \(g\) of \(n\)”) the set of functions

\[
\Omega(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq cg(n) \leq f(n) \text{ for all } n \geq n_0 \}.
\]

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**little oh**

o-notation

The asymptotic upper bound provided by \(O\)-notation may or may not be asymptotically tight. The bound \(2n^2 = O(n^2)\) is asymptotically tight, but the bound \(2n = O(n^2)\) is not. We use \(o\)-notation to denote an upper bound that is not asymptotically tight. We formally define \(o(g(n))\) (“little-oh of \(g\) of \(n\)”) as the set

\[
o(g(n)) = \{ f(n) : \text{for any positive constant } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \leq f(n) < cg(n) \text{ for all } n \geq n_0 \}.
\]

For example, \(2n = o(n^2)\), but \(2n^2 \neq o(n^2)\).

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**Big Theta**

Θ-notation

In Chapter 2, we found that the worst-case running time of insertion sort is \(T(n) = \Theta(n^2)\). Let us define what this notation means. For a given function \(g(n)\), we denote by \(\Theta(g(n))\) the set of functions

\[
\Theta(g(n)) = \{ f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0 \}.
\]

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**little omega**

ω-notation

By analogy, \(\omega\)-notation is to \(\Omega\)-notation as \(o\)-notation is to \(O\)-notation. We use \(\omega\)-notation to denote a lower bound that is not asymptotically tight. One way to define it is by

\[
f(n) \in \omega(g(n)) \text{ if and only if } g(n) \in o(f(n)).
\]

Formally, however, we define \(\omega(g(n))\) (“little-omega of \(g\) of \(n\)”) as the set

\[
\omega(g(n)) = \{ f(n) : \text{for any positive constant } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \leq cg(n) < f(n) \text{ for all } n \geq n_0 \}.
\]

For example, \(n^2/2 = \omega(n)\), but \(n^2/2 \neq \omega(n^2)\).