Introduction

Insertion Sort
Merge Sort

Insertion Sort

Figure 2.2: Illustrating Insertion Sort

Insertion Sort Analysis

Merge

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Merge Sort

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Insertion Sort

\[
\begin{align*}
\text{INSERTION-SORT} & (A) \\
1 & \text{for } j = 2 \text{ to } A.\text{length} \\
2 & \quad key = A[j] \\
3 & \quad // Insert } A[j] \text{ into the sorted sequence } A[1..j-1]. \\
4 & \quad i = j - 1 \\
5 & \quad \text{while } i > 0 \text{ and } A[i] > key \\
6 & \quad \quad A[i + 1] = A[i] \\
7 & \quad \quad i = i - 1 \\
8 & \quad A[i + 1] = key
\end{align*}
\]

Figure 2.2: Illustrating Insertion Sort

Figure 2.3A: Illustrating Merge, Part 1
Figure 2.3B: Illustrating Merge, Part 2
Figure 2.4: Illustrating Merge Sort
Figure 2.5A: Analyzing Merge Sort, Part 1
Figure 2.5B: Analyzing Merge Sort, Part 2
Insertion Sort Analysis

**Insertion-Sort(A)**

<table>
<thead>
<tr>
<th>cost</th>
<th>times</th>
</tr>
</thead>
<tbody>
<tr>
<td>[c_1 \times n]</td>
<td>(n)</td>
</tr>
<tr>
<td>[c_2 \times n - 1]</td>
<td>(n - 1)</td>
</tr>
<tr>
<td>(0)</td>
<td>(n - 1)</td>
</tr>
<tr>
<td>(c_4 \times n)</td>
<td>(n)</td>
</tr>
<tr>
<td>(c_5 \times \sum_{j=2}^{n} t_j)</td>
<td>[\sum_{j=2}^{n} (t_j - 1)]</td>
</tr>
<tr>
<td>(c_6 \times i = i - 1)</td>
<td>(c_7 \times \sum_{j=2}^{n} (t_j - 1))</td>
</tr>
<tr>
<td>(c_8 \times A[i + 1] = key)</td>
<td>(n - 1)</td>
</tr>
</tbody>
</table>

Merge

**MERGE(A, p, q, r)**

1. \(n_1 = q - p + 1\)
2. \(n_2 = r - q\)
3. Let \(L[1..n_1 + 1]\) and \(R[1..n_2 + 1]\) be new arrays
4. for \(i = 1\) to \(n_1\)
5. \(L[i] = A[p + i - 1]\)
6. for \(j = 1\) to \(n_2\)
7. \(R[j] = A[q + j]\)
8. \(L[n_1 + 1] = \infty\)
9. \(R[n_2 + 1] = \infty\)
10. \(i = 1\)
11. \(j = 1\)
12. for \(k = p\) to \(r\)
13. if \(L[i] \leq R[j]\)
14. \(A[k] = L[i]\)
15. \(i = i + 1\)
16. else \(A[k] = R[j]\)
17. \(j = j + 1\)

Figure 2.3A: Illustrating Merge, Part 1

Figure 2.3B: Illustrating Merge, Part 2
Merge Sort

\textsc{merge-sort}(A, p, r)
1 \textbf{if} \ p < r
2 \hspace{1em} q = \lceil (p + r)/2 \rceil
3 \hspace{1em} \textsc{merge-sort}(A, p, q)
4 \hspace{1em} \textsc{merge-sort}(A, q + 1, r)
5 \hspace{1em} \textsc{merge}(A, p, q, r)

Figure 2.4: Illustrating Merge Sort

sorted sequence

1 2 2 3 4 5 6 7

merge

2 4 5 7

merge

2 5

merge

4 7

merge

1 3

merge

2 6

merge

5 2

merge

4 7

merge

1 3

merge

2 6

initial sequence

Figure 2.5A: Analyzing Merge Sort, Part 1

\[ T(n) \]
\begin{align*}
\text{cn} & \quad T(n/2) \\
T(n/2) & \quad T(n/2) \\
\text{cn/2} & \quad T(n/4) \\
\text{cn/2} & \quad T(n/4) \\
\text{cn/2} & \quad T(n/4) \\
\text{cn/2} & \quad T(n/4) \\
\end{align*}

Figure 2.5B: Analyzing Merge Sort, Part 2

\[ \text{Total: } cn \log n + cn \]