In the Cookie Problem, cookies are randomly placed in a $n \times n$ grid. The cookie monster starts at the lower left corner (which always has no cookies). The cookie monster can go left, right, up, or down one square at a time (not diagonally). The goal of the cookie monster is to eat $m$ cookies.

1. (20 pts.) Display the whole state space for the $2 \times 2$ Cookie Problem displayed below. The cookie monster starts in the lower left square.

\[
\begin{array}{cc}
1 & 2 \\
0 & 3 \\
\end{array}
\]

Hint: there are 18 states and two edges from each state. Note that a state consists of the location of the cookie monster and the cookies remaining in the grid. Once the cookies are gone, the cookie monster can still move around the grid.

2. (20 pts.) For the Cookie Problem, estimate the number of states in the state space for an $n \times n$ grid. Assume that the cookie monster can eat up all the cookies. Provide a lower bound and an upper bound. Hint: It’s much much bigger than $n^2$.

3. (20 pts.) For the Cookie Problem above, show the sequence of states that are visited for breadth-first search (BFS) and iterative deepening (ID). The cookie monster starts in the lower left square. Let the goal state be to eat all 6 cookies. A state is considered visited when it is tested for being a goal state.

Assume that BFS uses multiple path pruning, that is, no state is expanded more than once (see Section 3.7.2).

Assume that ID uses cycle checking, that is, no state appears twice in a path (see Section 3.7.1).

4. Consider the following Cookie problem. The goal is to eat four cookies. As usual, the cookie monster starts in the lower left square.

\[
\begin{array}{ccc}
0 & 2 & 1 \\
1 & 0 & 0 \\
0 & 0 & 2 \\
\end{array}
\]

Define a heuristic for the Cookie problem as follows. Let $c$ be the goal number of cookies.

For a given state, let $b$ be the number of cookies eaten so far. Let $a_1$ be the maximum number of cookies in a square that can be reached in 1 move. Let $a_2$ be the maximum number of cookies in a square that can be reached in 1 or 2 moves. Let $a_3$ be the
maximum number of cookies in a square that can be reached in 1 or 2 or 3 moves.
Define $a_4$, $a_5$, and so on similarly.

Now the value of the heuristic is $i$ if $b + a_1 + \ldots + a_{i-1} < c$ and $b + a_1 + \ldots + a_i \geq c$.
That is, $i - 1$ moves is clearly not enough to eat $c - b$ more cookies, while $i$ moves might be.

For example, the initial state has a heuristic value of 3. First, we know that $b = 0$, $c = 4$, $a_1 = 1$, $a_2 = 2$, $a_3 = 2$, $a_4 = 2$, and so on. The result is 3 because $b + a_1 + a_2 < c$ and $b + a_1 + a_2 + a_3 \geq c$.

(a) (20 pts.) Prove that the heuristic is admissible. That is, it never overestimates the number of moves to a goal state. That is, why are $i - 1$ moves not enough to eat $c - b$ more cookies?

(b) (20 pts.) Show the order in which A* search would visit the states. In case of ties, prefer states generated first. When generating the successors of a state, generate them in the order “move right”, “move up”, “move left”, and “move down”.