Making Decisions

Introduction

- Planning under uncertainty should address:
  - The world is nondeterministic.
  - Actions are not certain to succeed.
  - Many events are outside of the agent's control.
  - An agent doesn't know the complete state of the world.
  - An agent has multiple goals. Some could be partially satisfied.
- Use *probability* to represent ignorance.
- Use *utility* to represent preference.
- A *rational agent maximizes expected utility*.

Utility

Definitions

- An agent specifies *preferences* on *outcomes* (notation $o_1, o_2, \ldots$).
- $o_1 \succ o_2$ means outcome o_1 is *preferred* over o_2.
  $o_1 \sim o_2$ means *indifference* between o_1 and o_2.
- A *lottery* is a probability distribution over outcomes, written as

  $[p_1 : o_1, p_2 : o_2, \ldots, p_k : o_k]$

  where the p_is are probabilities that sum to 1.
- The lottery specifies that outcome o_i occurs with probability p_i.
- The outcomes in a lottery can be other lotteries.

Utility

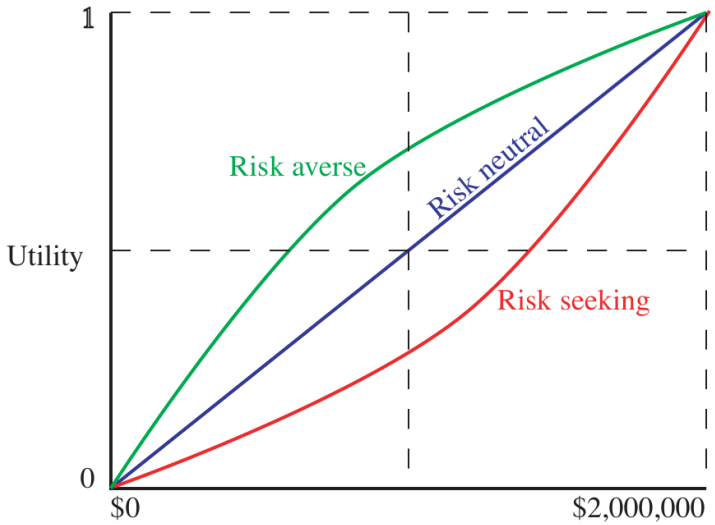
If an agent is *rational*, then there is a utility function u such that

- $o_i \succ o_j$ if and only if $u(o_i) > u(o_j)$ and
- u is linear with probabilities:

  $u([p_1 : o_1, p_2 : o_2, \ldots, p_k : o_k]) = p_1 u(o_1) + p_2 u(o_2) + \ldots + p_k u(o_k)$

Rational is defined as satisfying the following properties of preferences (see book):
Completeness, Transitivity, Monotonicity, Decomposability, Continuity, Substitutability

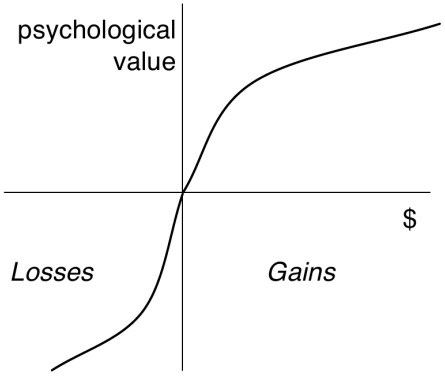
Utility of Money



Money Preferences

- What would you prefer?
  - A. $1,000,000
  - B. lottery [0.5:$0, 0.5:$2,000,000]
- What would you prefer?
  - C. $1m – one million dollars
  - D. lottery [0.10:$2.5m, 0.89:$1m, 0.01:$0]
- What would you prefer?
  - E. lottery [0.11:$1m, 0.89:$0]
  - F. lottery [0.10:$2.5m, 0.9:$0]

People's Typical Value of Money



- Loss aversion is greater than gain utility.
- If all choices bad, behavior is risk seeking.

Utility and Time

- Would you prefer $1000 today or $1000 next year?
- How would you compare the following sequences of rewards (per day, per week, per year)?
  - A. $1000000, $0, $0, $0, $0, $0, ...
  - B. $1000, $1000, $1000, $1000, $1000, ...
  - C. $1000, $0, $0, $0, $0, ...
  - D. $1, $1, $1, $1, $1, ...
  - E. $1, $2, $3, $4, $5, ...

Rewards

Suppose the agent receives a sequence of rewards $r_1, r_2, r_3, r_4, \ldots$ in time. What utility should be assigned?

- total reward $V = \sum_{i=1}^{n} r_i$
- average reward $V = \sum_{i=1}^{n} r_i/n$
- discounted reward $V = r_1 + \gamma r_2 + \gamma^2 r_3 + \gamma^3 r_4 + \ldots = \sum_{i=1}^{n} \gamma^{i-1} r_i$ where $0 < \gamma < 1$ is the *discount factor.*¿G
- Why discount? A reward now is worth more than the same reward later.

Properties of Discounts

- With an infinite horizon, the total reward can be infinite: $1, $1, $1, ... and average reward can be infinite: $1, $2, $3, ...
  It is hard to compare one infinite with another.
- Discounted reward is always finite.
- Note that $1 + \gamma + \gamma^2 + \gamma^3 + \ldots = 1/(1-\gamma)$
- If r_{min} and r_{max} are the minimum and maximum rewards, discounted reward is between $r_{min}/(1-\gamma)$ and $r_{max}/(1-\gamma)$

Decision Networks

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Assumptions

Decision theory for intelligent agents assumes:

- Agents know what actions they can carry out.
- The effects of actions are described as probabilities over outcomes.
- An agent's preferences are expressed by utilities of outcomes.

Decision theory specifies how to trade off the desirability and probabilities of different outcomes from different actions.

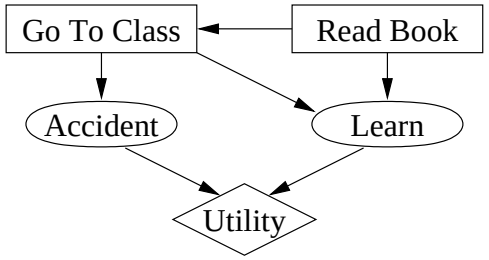
Single-State Decision Networks

Single-stage decision networks extend belief networks. There are three types of variables:

- *Random variables* are belief network variables. Value depends on probability table.
- *Decision variables* whose values are actions. The agent chooses a value for each decision variable. Drawn as rectangles.
- *Utility node*. Its parents are the variables on which the utility depends. Drawn as a diamond.

Solve by calculating the *expected utility* of each decision assignment.

Example



G	$A{=}T$	$A{=}F$
T	0.001	0.999
F	0.0001	0.9999

G	R	$L{=}T$	$L{=}F$
T	T	0.9	0.1
T	F	0.5	0.5
F	T	0.5	0.5
F	F	0.1	0.9

A	L	U
T	T	-99
T	F	-100
F	T	1
F	F	0

Expected Utility

- A possible world ω assigns a value to each random/decision variable.
- Let the decision(s) be δ.
- Expected utility of $\delta = \sum_\omega P(\omega \mid \delta) * U(\omega)$
- Expected utility of $G = T, R = F$ is 0.4.

G	R	A	L	P	u
T	F	T	T	0.0005	-99
T	F	T	F	0.0005	-100
T	F	F	T	0.4995	1
T	F	F	F	0.4995	0

- Variable elimination can be used for expected utility. Eliminate vars below decision vars, then select best decisions.

Sum Out One Random Variable

G	A	P
T	T	0.001
T	F	0.999
F	T	0.0001
F	F	0.9999

$*$

A	L	U
T	T	-99
T	F	-100
F	T	1
F	F	0

Multiplying and summing out A yields:

G	L	$ExpectedUtility$
T	T	$0.001 * (-99) + 0.999 * (1) = 0.9$
T	F	$0.001 * (-100) + 0.999 * (0) = -0.1$
F	T	$0.0001 * (-99) + 0.9999 * (1) = 0.99$
F	F	$0.0001 * (-100) + 0.9999 * (0) = -0.01$

Sum Out The Other Random Variable

G	R	L	P
T	T	T	0.9
T	T	F	0.1
T	F	T	0.5
T	F	F	0.5
F	T	T	0.5
F	T	F	0.5
F	F	T	0.1
F	F	F	0.9

Optimal: go to class and read the book.

$*$

G	L	EU
T	T	0.9
T	F	-0.1
F	T	0.99
F	F	-0.01

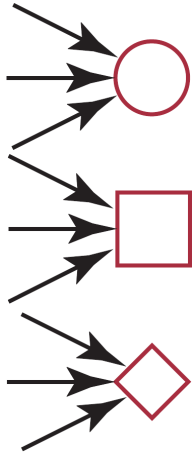
Multiply and sum out L:

G	R	$ExpectedUtility$
T	T	$.9(.9) + .1(-.1) = .8$
T	F	$.5(.9) + .5(-.1) = .4$
F	T	$.5(.99) + .5(-.01) = .49$
F	F	$.1(.99) + .9(-.01) = .09$

Sequential Decision Networks

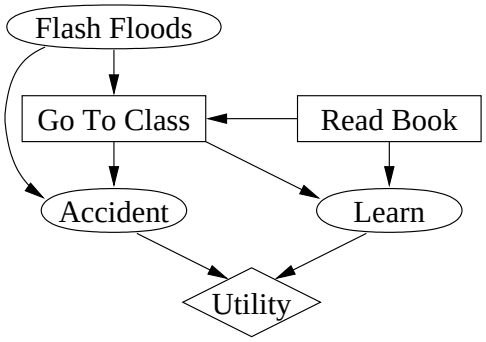
- Typically, an agent will base its decisions on information it knows.
- A *sequential decision problem* consists of a sequence of decision variables $D_1, \ldots, D_n$.
- Each D_i has variables $parents(D_i)$, whose value will be known at the time decision D_i is made.
- A *policy* specifies what an agent should do under each circumstance. A decision function for D_i maps from values of $parents(D_i)$ to a value for D_i.
- Variable elimination: Eliminate vars below D_i, then eliminate D_i by selecting best policy.

Variable Types



- A random variable is drawn as an ellipse. Edges into the node represent probabilistic dependence.
- A decision variable is drawn as an rectangle. Edges into the node represent information available when the decision is made.
- A utility node is drawn as a diamond. Edges into the node represent variables that the utility depends on.

Example



FF=T	FF=F
0.01	0.99

FF	G	A=T	A=F
T	T	0.01	0.99
T	F	0.0001	0.9999
F	T	0.001	0.999
F	F	0.0001	0.9999

Start Sum Out

FF	G	A	P
T	T	T	0.01
T	T	F	0.99
T	F	T	0.0001
T	F	F	0.9999
F	T	T	0.001
F	T	F	0.999
F	F	T	0.0001
F	F	F	0.9999

$*$

A	L	U
T	T	-99
T	F	-100
F	T	1
F	F	0

Finish Sum Out

Multiplying and summing out A yields:

FF	G	L	$ExpectedUtility$
T	T	T	$.01(-99) + .99(1) = 0$
T	T	F	$.01(-100) + .99(0) = -1$
T	F	T	$.0001(-99) + .9999(1) = .99$
T	F	F	$.0001(-100) + .9999(0) = -.01$
F	T	T	$.001(-99) + .999(1) = .9$
F	T	F	$.001(-100) + .999(0) = -.1$
F	F	T	$.0001(-99) + .9999(1) = .99$
F	F	F	$.0001(-100) + .9999(0) = -.01$

Start Other Sum Out

G	R	L	P
T	T	T	0.9
T	T	F	0.1
T	F	T	0.5
T	F	F	0.5
F	T	T	0.5
F	T	F	0.5
F	F	T	0.1
F	F	F	0.9

$*$

FF	G	L	EU
T	T	T	0.0
T	T	F	-1.0
T	F	T	0.99
T	F	F	-0.01
F	T	T	0.9
F	T	F	-0.1
F	F	T	0.99
F	F	F	-0.01

Finish Sum Out

Multiplying and summing out L yields:

FF	G	R	$ExpectedUtility$
T	T	T	$0.9 * (0) + 0.1 * (-1) = -0.1$
T	T	F	$0.5 * (0) + 0.5 * (-1) = -0.5$
T	F	T	$0.5 * (0.99) + 0.5 * (-0.01) = 0.49$
T	F	F	$0.1 * (0.99) + 0.9 * (-0.01) = 0.09$
F	T	T	$0.9 * (0.9) + 0.1 * (-0.1) = 0.8$
F	T	F	$0.5 * (0.9) + 0.5 * (-0.1) = 0.4$
F	F	T	$0.5 * (0.99) + 0.5 * (-0.01) = 0.49$
F	F	F	$0.1 * (0.99) + 0.9 * (-0.01) = 0.09$

Finish Sum Out

Choose values for G that maximize expected utility.

FF	G	R	$ExpectedUtility$
T	F	T	0.49
T	F	F	0.09
F	T	T	0.8
F	T	F	0.4

Policy for G:
Map $FF{=}T$ to $G{=}F$, and $FF{=}F$ to $G{=}T$.
Don't go to class when there are flash floods.

Policy for R: Choose $R = T$.

Markov Decision Processes

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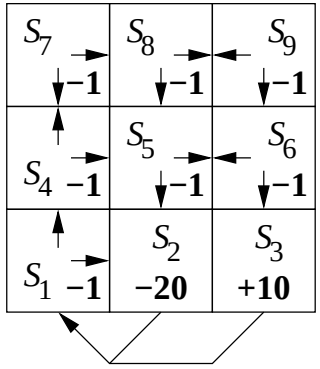
An MDP Game



This game has nine squares named S_1 to S_9.

The agent starts at S_1.

The arrows show what moves are allowed.

In S_2 and S_3, the agent can only move to S_1.

Each move costs 1 unit (a *reward* of -1), except S_2 (reward -20) and S_3 (reward 10).

90% of the time, the chosen move succeeds.
10% of the time, the other move happens.

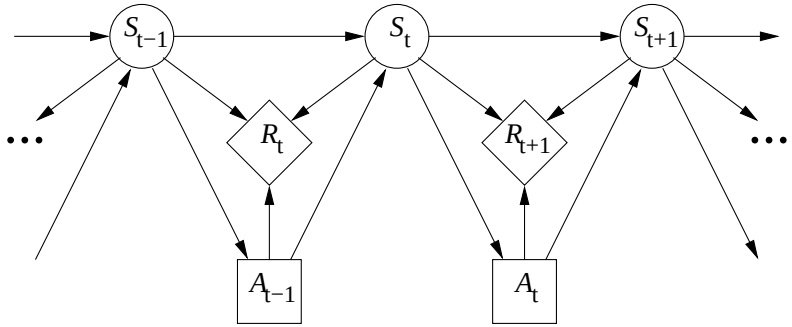
Markov Decision Process

A *Markov decision process* (MDP) consists of:

- S, a set of states
- A, a set of actions
- $P : S \times S \times A \to [0, 1]$ is a probabilistic transition function $P(s' \mid s, a)$, the probability that action a will move from state s to state s'.
- $R : S \times A \times S \to \Re$ is the reward function. $R(s, a, s')$ is the reward when action a moves from state s to state s'.

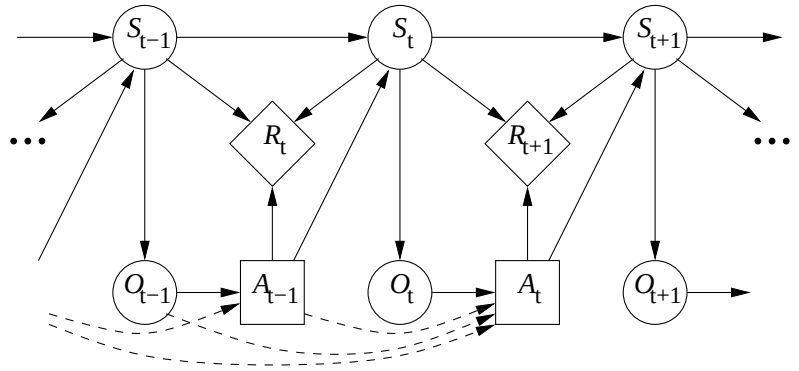
The next state depends only on the current state and action. The previous states and actions do not add any more information.

Decision Network: Fully Observable MDP



In a *fully observable* MDP, the agent knows the current state.

Decision Network: Partially Observable MDP



In a *partially observable* MDP (POMDP), the agent only sees evidence of the current state: current observations, previous actions/ observations, etc.

Policies

- A *policy* is a function: $\pi : S \rightarrow A$
- Given a state s, $\pi(s)$ specifies the action.
- An *optimal policy* is the one with the maximum value (expected discounted reward).
- For a sequence of rewards $r_1, r_2, r_3 \ldots$, the *discounted reward* is:

  $V = r_1 + \gamma r_2 + \gamma^2 r_3 + \ldots = \sum_{i=1}^{\infty} \gamma^{i-1} r_i$

  where $0 < \gamma < 1$ is the *discount factor*.
- For a fully-observable MDP with stationary dynamics and rewards and with infinite or indefinite horizon, there is always an optimal stationary policy.

The Value of a Policy

- $Q^\pi(s, a)$ is the expected value of doing action a in state s, then following policy π.
- $V^\pi(s)$ is the expected value of following policy π in state s.
- Q^π and V^π can be recursively defined:

  $V^\pi(s) = Q^\pi(s, \pi(s))$

  $Q^\pi(s, a) = \sum_{s'} P(s'|s, a)(R(s, a, s') + \gamma V^\pi(s'))$
- π is an optimal stationary policy if and only if:

  $\pi(s) = \arg\max_a Q^\pi(s, a)$
- Value iteration is an algorithm for optimizing π.

Value Iteration for FOMDPs

Procedure *Value-Iteration*(S, A, P, R, ϵ)
  Inputs: states, actions, transition function,
    reward function, convergence threshold
  $V \leftarrow$ array with $|S|$ values
  $Q \leftarrow$ 2D array with $|S| \times |A|$ values
  repeat
    for each state $s \in S$
      for each action $a \in A$
        $Q[s, a] \leftarrow \sum_{s'} P(s'|s, a)(R(s, a, s') + \gamma V(s'))$
      $V[s] \leftarrow \max_a Q[s, a]$
  until no value in V changes by more than ϵ
  return V, Q

Idea of Value Iteration

- Assume V, Q initialized to zeroes.
- After one loop, V is the maximum expected immediate reward.
- After k loops, V is the maximum expected discounted reward, looking k steps ahead.
- It converges exponentially fast (in k) to the optimal values. The error is proportional to γ^k (assuming $\gamma > 0.5$).
- Optimal policy can be easily computed from Q or V.

Back to the MDP Game

Using $\gamma = 0.99$, $\epsilon = 0.001$, here are the Q values from value iteration.