Features

Basics

- Consider problems where a solution is a single state (Sudoku, schedule).
- Instead of treating each state like a black box, one can reason about the *features* of a state.
- Each feature is mapped to a *variable*, which has a set of possible values, its *domain*.
- A *possible world* is an assignment of values to the variables. A valid possible world is a *model*.
- A *hard constraint* specifies valid combinations of values for, typically, a small subset of the variables.
- A *soft constraint* specifies the costs of different combinations of values.

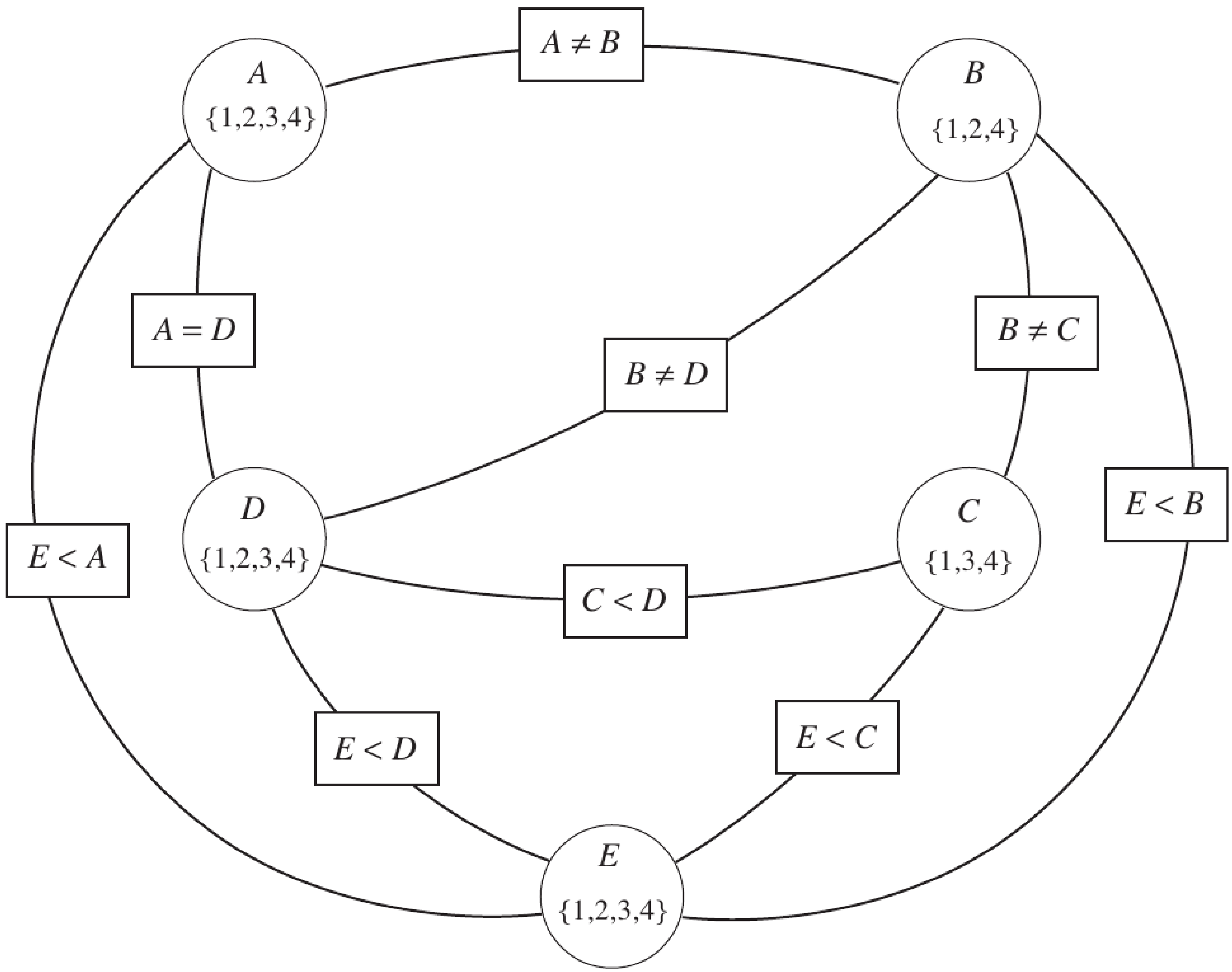
Constraint Satisfaction Problems

- A Constraint Satisfaction Problems (CSP) consist of a set of variables, a domain for each variable, and a set of (hard) constraints.
- One way to solve CSPs is to use *arc consistency*.
  - Each variable X starts with all of its possible values.
  - A possible value x can be eliminated if arc inconsistent with a constraint c.
  - This means no combination of possible values allowed by c includes $X = x$.
- To solve CSPs, combine arc consistency with domain splitting, trying all values of a variable.

Generalized Arc Consistency

Procedure *GAC*(V, C, D)
  Inputs: V: a set of variables
    C: a set of constraints
    D_X: a set of values for each variable X
  Output: an arc-consistent D
  repeat until no more changes are made
    for each constraint $c \in C$
      for each variable X used by c
        for each value $x \in D_X$
          if $X = x$ is not valid for c and D
            remove x from D_X
  return D

Example CSP


A
$\{1,2,3,4\}$
B
$\{1,2,4\}$
C
$\{1,3,4\}$
D
$\{1,2,3,4\}$
E
$\{1,2,3,4\}$
$A \neq B$
$A = D$
$B \neq D$
$B \neq C$
$E < A$
$C < D$
$E < B$
$E < D$
$E < C$

Domain Splitting

Procedure *DS*(V, C, D)
  Inputs: V: a set of variables
     C: a set of constraints
     D_X: a set of values for each variable X
  Output: null or a solution D'
  $D \leftarrow$ *GAC*(V, C, D)
  if some $D_X = \emptyset$ return null
  if every D_X has one value return D
  $X \leftarrow$ some variable where $|D_X| > 1$
  for each value $x \in X$
    $D' \leftarrow$ *DS*(V, C, copy of D with $D'_X = \{x\}$)
    if $D' \neq$ null return D'
  return null

Local Search

Local search methods try to improve an assignment to the variables by taking a series of small steps. The goal is to to efficiently find good solutions.

Procedure *Local-Search*(V, C)
  Inputs: V: a set of variables
      C: a set of constraints
  Output: a solution A
  $A \leftarrow$ an assignment of variables to values
  while A does not satisfy C
    $A \leftarrow$ choose a "neighbor" of A
  return A

Iterative Improvement

- An *evaluation function* provides the value of each state.
  - For CSPs, one could use the number of constraints that are not satisfied.
- *Hill climbing* selects the best neighbor.
- For continuous variables, *gradient descent* changes all the variables based on partial deriviatives.

Local Optima

The problem with iterative improvement is finding states that are locally optimal, but far from globally optimal.

Randomization

Use *randomization* to avoid local optima.

- Random restart: repeatedly start local search with different assignments.
- Random walk: interleave random steps with improvement steps.
- Tabu list: avoid recently used variable-value assignments to try to avoid cycles.
- Simulated annealing: Start local search with mostly random steps and gradually increase the proportion of improvement steps.

Genetic Algorithms

Genetic algorithms is a popular local search method, inspired by natural selection.

- Population: Instead of storing one assignment (individual) at a time, store many individuals.
- Mutation: A new individual can be generated by random small steps from an old individual.
- Crossover: A new individual can be generated by combining the variable assignments of two old individuals.
- Random Selection: Select the new population using the evaluation function combined with some randomness.