Multiple Agents

Why can't we all just get along? (Rodney King)

Introduction

Assumptions

- Each agent can act autonomously.
- Each agent has its own information about the world.
- Each agent can have its own utility function.
- A *mechanism* specifies how the actions of the agents lead to outcomes, e.g., rules of chess.
- A rational agent acts *strategically*; its actions are based on utility.
- *Nature* can be defined as an agent with no utility and no strategy.

Definitions

- Agents can be *fully cooperative*; they share the same utility.
- Agents can be *fully competitive*; they have the opposite utility.
- In a *zero-sum game*, the sum of the utilities for the agents is zero for every outcome.
- *Game theory* studies what agents should do in a multi-agent setting.

Games

Normal Form of a Game

The *strategic form* or *normal form of a game* contains:

- a finite set I of agents, $\{1, \ldots, n\}$.
- a set of strategies for each agent.
- A *strategy profile* $s = (s_1, \ldots, s_n)$ means that agent i follows strategy s_i.
- utility functions $u_i(s)$ gives the expected utility for agent i when all agents follow strategy profile s.
- An *outcome* is produced when all the agents follow a strategy profile.

Simple Example: Rock-Paper-Scissors

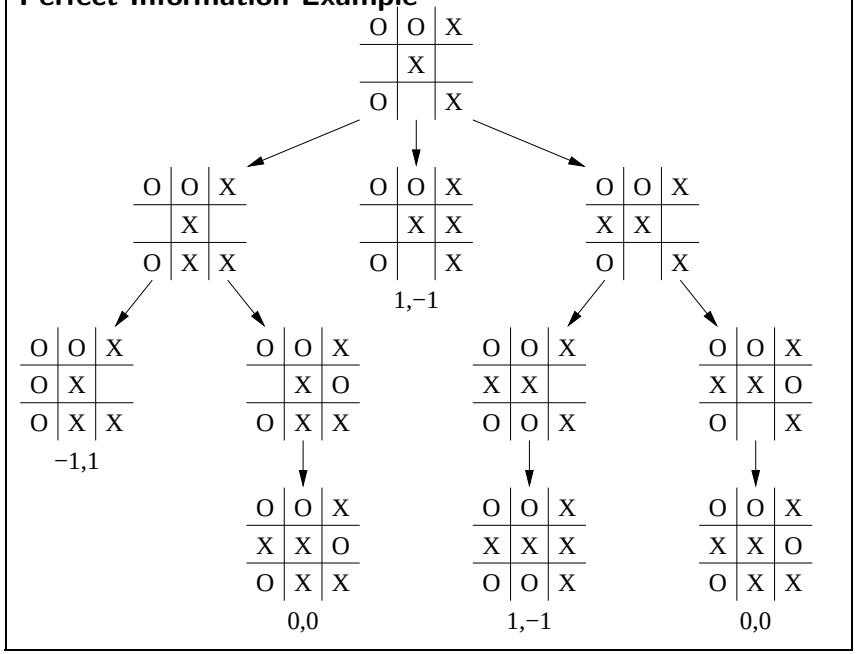
- In the game of rock-paper-scissors, there are two agents, each choosing one of three actions {rock,paper, scissors}.
- For each combinations of actions, a *payoff matrix* can be used to specify the utilities.

		Agent 2		
		rock	paper	scissors
Agent 1	rock	$0, 0$	$-1, 1$	$1, -1$
	paper	$1, -1$	$0, 0$	$-1, 1$
	scissors	$-1, 1$	$1, -1$	$0, 0$

Extensive Form of a Game

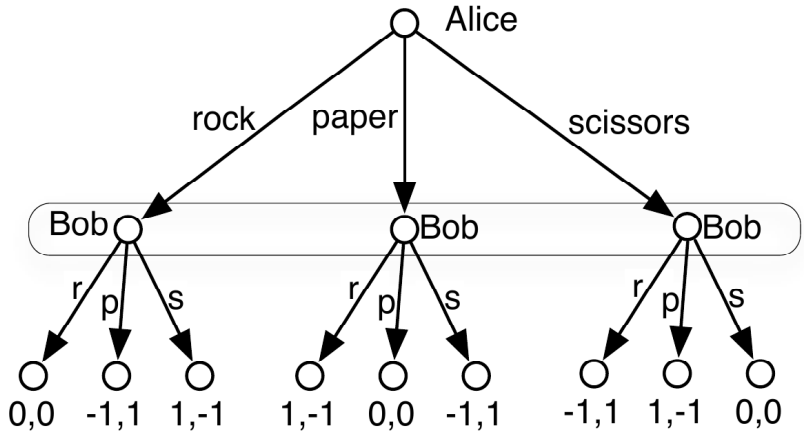
- The *extensive form of a game* or a *game tree* is a finite tree where the nodes are states and the edges are actions.
- Each internal node is *controlled* by an particular agent.
- Each edge out of a node controlled by agent i corresponds to an action for agent i.
- Each node controlled by nature has a probability distribution over its children.
- The leaves represent final outcomes and are labeled with a utility for each agent.

Perfect Information Example



Imperfect Information Example

An agent cannot distinguish the nodes in an *information set*.

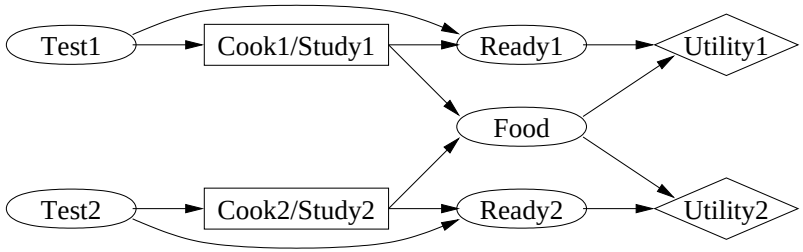


Multiagent Decision Networks

- A multiagent decision network is a factored representation of a multiagent decision problem.
- Each decision node is labeled with an agent that makes the decision for the node.
- Each agent has a utility node.
- As with a decision network, the parents of a decision node are the information for making the decision.

Example Multiagent Decision Network

The scenario is that two roommates each might have a test to study for, choosing then to study or cook some food for both of them to eat.

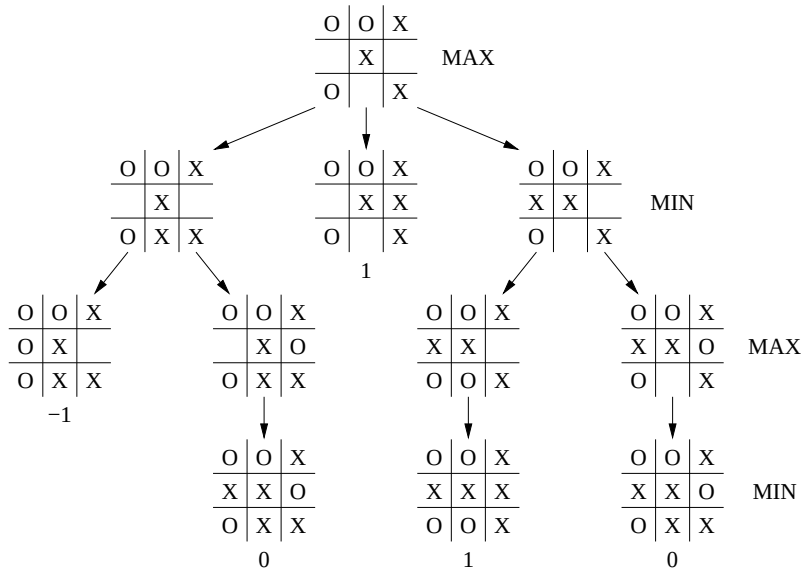


Two-Agent Zero-Sum Games

Two-Agent Zero-Sum Games

- A *two-agent zero-sum game* is when a positive reward for one agent is an equally negative reward for the other agent.
- The utility can be characterized by a single number that one agent is trying to maximize and the other agent is trying to minimize.
- Having a single value for a two-agent zero-sum game leads to a *minimax* strategy.
- Each node is either a MAX node, if controlled by the maximizing agent, or a MIN node if controlled by the minimizing agent.
- Treat agent currently in control (whose turn it is to move) as MAX.

Minimax Example



Minimax Procedure

Procedure *Minimax*(N)
  Inputs: a node in a game tree
  if N is a leaf node, then
    $v \leftarrow$ value of N
  else if N is a MAX node
    $v \leftarrow -\infty$
    for each child C of N
      $v \leftarrow$ maximum of v and *Minimax*(C)
  else if N is a MIN node
    $v \leftarrow +\infty$
    for each child C of N
      $v \leftarrow$ minimum of v and *Minimax*(C)
  return v

Evaluation Functions

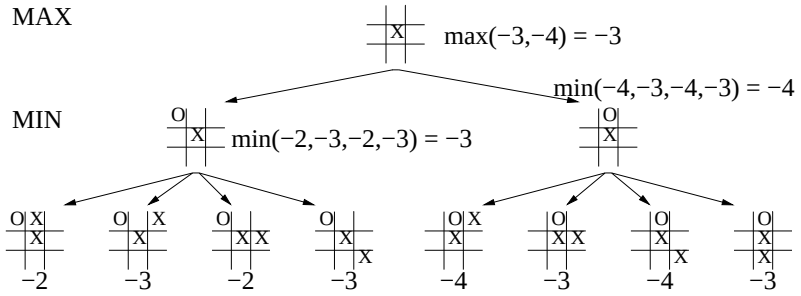
The game tree of many games is too large to search. An alternative is:

- Search as deeply as possible given time requirement.
- Use an *evaluation function* to estimate the values of nodes at the fringe.
- Use minimax to combine the values into an overall evaluation.

Example

evaluation = +10 if MAX has 3 in line
−10 if MIN has 3 in line
+1 for each potential 3 in line for MAX
−1 for each potential 3 in line for MIN

MAX is O; MIN is X



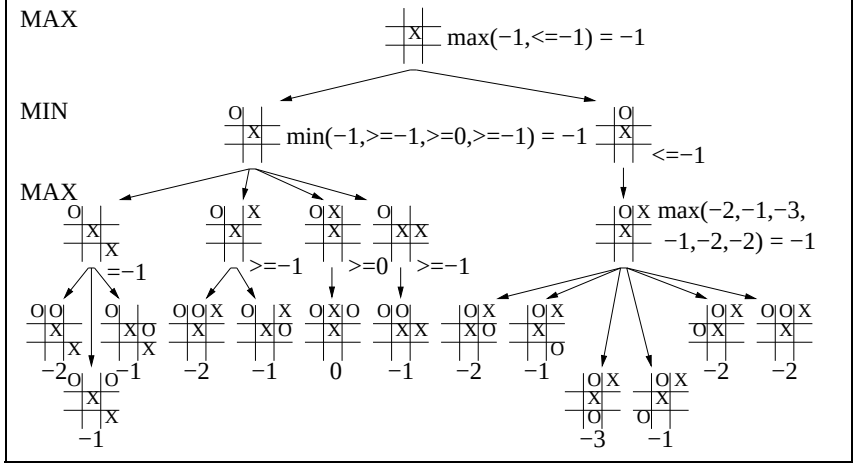
Minimax Procedure with Evaluation Function

Procedure *Minimax*(N, d, f)
  Inputs: game tree node, search depth, eval. fn.
  if N is a leaf node, then return value of N
  else if $d = 0$, then return $f(N)$
  else if N is a MAX node
    $v \leftarrow -\infty$
    for each child C of N
      $v \leftarrow$ maximum of v and *Minimax*$(C, d-1, f)$
  else if N is a MIN node
    $v \leftarrow +\infty$
    for each child C of N
      $v \leftarrow$ minimum of v and *Minimax*$(C, d-1, f)$
  return v

Idea of Alpha-Beta Pruning

- Alpha-beta pruning avoids search that won't change the minimax evaluation.
- Example: If MAX has a move with value 3, stop searching other moves known to be ≤ 3.
- General Principle: Consider a node N.
  $\alpha =$ largest v in MAX ancestors of N.
  $\beta =$ smallest v in MIN ancestors of N.
  If $\alpha \geq \beta$, processing N cannot change eval.
- Proof: Let $v = N$'s minimax value.
  $\alpha \geq \beta$ implies $\alpha \geq v$ or $v \geq \beta$.
  $\alpha \geq v$ implies v can't change ancestor with α.
  $v \geq \beta$ implies v can't change ancestor with β.
  This implies v cannot propagate to the top.

Example



Alpha-Beta Procedure with Eval Function

Procedure *Alpha-Beta*(N, d, f, α, β)
  if N is a leaf node, then return value of N
  else if $d = 0$, then return $f(N)$
  else if N is a MAX node
    for each child C of N
      $\alpha \leftarrow$ max(α, *Alpha-Beta*$(C, d-1, f, \alpha, \beta)$)
      if $\alpha \geq \beta$ return α
  else if N is a MIN node
    for each child C of N
      $\beta \leftarrow$ min(β, *Alpha-Beta*$(C, d-1, f, \alpha, \beta)$)
      if $\alpha \geq \beta$ return β
  if N is a MAX node, then return α, else return β

Performance of Minimax and Alpha-Beta

- $b =$ branching factor
  $d =$ depth of search
- Minimax visits every state from level 0 to d.
  $\sum_{i=0}^{d} b^i = \frac{b^{d+1}-1}{b-1} \in O(b^d)$
- Alpha-Beta visits as few as $\Omega(b^{d/2})$ states.
  Depends on a good ordering of children.
  Actual programs approach the minimum bound.
- Alpha-beta pruning allows programs to look ahead nearly twice as many moves as minimax.

Other Issues

- Horizon problem
- Quiescence
- Data bases of openings and end games
- Games of chance
  - *Expectiminimax* extends minimax to include chance nodes in the game tree.
  - Example: Roll of dice in Backgammon
  - Evaluate chance node by summing probability times child's value.

Partially Observable Multiagent Reasoning

Example of Partially Observable Multiagents

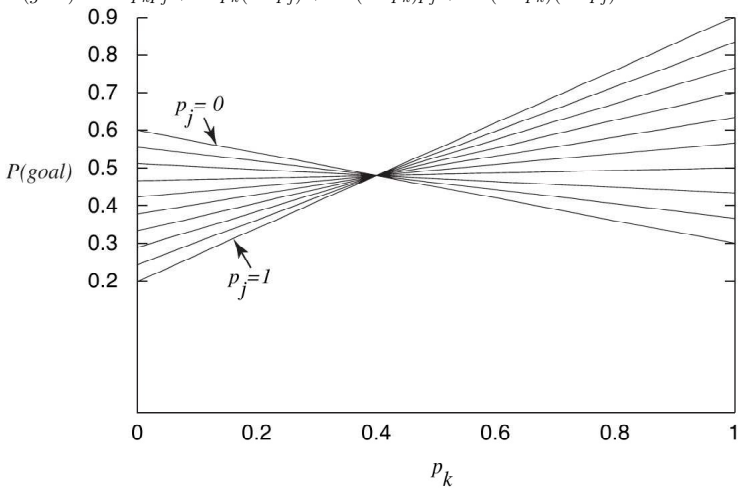
What should the kicker and goalie do?

		goalie: left	goalie: right
kicker	left	0.6	0.2
	right	0.3	0.9

Example of Partially Observable Multiagents

p_k = kicker kicks right, p_j = goalie jumps right

$P(goal) = 0.9p_kp_j + 0.3p_k(1-p_j) + 0.2(1-p_k)p_j + 0.6(1-p_k)(1-p_j)$



Strategy Profiles

- Assume a general n-player game,
- A *strategy* for an agent is a probability distribution over the actions for this agent.
- A strategy profile is an assignment of a strategy to each agent.
- A *strategy profile* s has a utility for each agent. Let $utility(s, i)$ be the utility of strategy profile s for agent i.
- If s is a strategy profile:
  s_i is the strategy of agent i in s,
  s_{-i} is the set of strategies of the other agents.
  Thus s is $s_i s_{-i}$

Nash Equilibriums

- s_i is a best response to s_{-i} if for all other strategies s'_i for agent i:
  $utility(s_i s_{-i}, i) \geq utility(s'_i s_{-i}, i)$
- A strategy profile s is a *Nash equilibrium* if, for each agent i, strategy s_i is a best response to s_{-i}. A Nash equilibrium is a strategy profile such that no agent can be better by changing its strategy.
- Theorem [Nash, 1950] Every finite game has at least one Nash equilibrium.
- In the soccer example, $p_k = 0.4$ and $p_j = 0.3$ is a Nash equilibrium.

Multiple Nash Equilibriums

- Hawk-Dove game ($D > R$):

		Agent 2: dove	Agent 2: hawk
Agent 1	dove	$R/2, R/2$	$0, R$
	hawk	$R, 0$	$-D, -D$

- Do what activity together?

		Agent 2: shopping	Agent 2: football
Agent 1	shopping	$2, 1$	$0, 0$
	football	$0, 0$	$1, 2$

Prisoners' Dilemma

Two prisoners for a crime can either stay silent or talk, providing evidence against the other prisoner. The payoff is the number of years in prison.

		Prisoner 2: silent	Prisoner 2: talk
Prisoner 1	silent	$-1, -1$	$-3, 0$
	talk	$0, 3$	$-2, -2$

Tragedy of the Commons

- There are 100 agents.
- There is a common environment shared by the agents.
- Each agent can choose do nothing with a 0 payoff, or do a selfish action that has a $+10$ payoff for the agent but includes a -1 payoff for all the other agents.
- If only one agent does a selfish action, that agent has a $+9$ payoff.
- If every agent does a selfish action, each agent gets a -90 payoff.

Computing Nash Equilibria

To compute Nash equilibria:

- Eliminate dominated strategies.
- Determine which actions will have non-zero probabilities. This is the support set.
- Determine the probability for the actions in the support set.

		Agent 2		
		x	y	z
Agent 1	a	$3,5$	$5,1$	$1,2$
	b	$1,1$	$2,9$	$6,4$
	c	$2,6$	$4,7$	$0,8$

Learning

- Issues: multiple equilibria, unknown utilities and outcomes for other agents
- Approach: Agent learns what is good for itself.
- Repeat:
  - Select action a using distribution P.
  - Do a and observe $payoff$.
  - Update $Q[a] \leftarrow Q[a] + \alpha(payoff - Q[a])$.
  - Find action a_{best} that maximizes Q.
  - Increase $P[a_{best}]$ and normalize P.
- If other agents have fixed, possibly stochastic, strategies, this algorithm converges near a best response (as long as all actions are tried occasionally).