

Combo	
11	<del>1136</del> 1135
12	<del>1138</del> 1134
13	<del>1137</del> 1135
14	<del>1136</del> 0
15	...
16	...
21	...

Combo	die2	val
11	1	1.0
12	2	1.0
13	3	...
14	4	...
15	5	...
16	6	...
21	...	...

evidenceList

Loop over evidenceList

create a Variable object

add it to the BN

create a Factor object with die1, die2  
and new variable

use set method 72 times on Factor

using the ~~the~~ right numbers

add Factor to BN

call observe method using  
current evidence bit

# HMM Probabilities

Probability

Conditional Independence

Bayesian Networks

Algorithms

Markov Models

Markov Chains

Hidden Markov Models

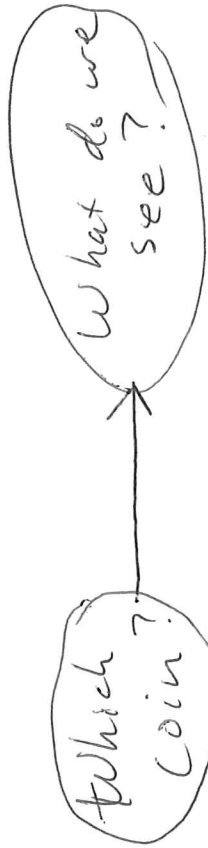
HMM

▷ Probabilities

- To compute  $P(S_i | O_0, \dots, O_i, \dots, O_k)$
- Compute variable elimination forward. This computes  $P(S_i, O_0, \dots, O_{i-1})$ .
- Compute variable elimination backward.
- This computes  $P(O_i, \dots, O_k, S_i)$ .
- $P(S_i, O_0, \dots, O_k) \propto P(S_i, O_0, \dots, O_{i-1}) * P(O_i, \dots, O_k | S_i)$
- To compute most probable sequence of states:
  - Viterbi algorithm.
  - Idea: Find most probable sequence to  $S_i$ .
  - Use that information to extend sequence by one state, and repeat.

# Three Coins Problem

## Bayesian Network



- Normal
- Two Heads
- Two Tails

- Head
- Tail

$P(w|ws=?)$

wc	$P(wc=?)$
N	$\frac{1}{3}$
TH	$\frac{1}{3}$
TT	$\frac{1}{3}$

wc	ws	$P(w ws=?)$
N	H	$\frac{1}{2}$
N	T	$\frac{1}{2}$
TH	H	0.0
TH	T	0.0
TT	H	0.0
TT	T	1.0

# Three Coins Joint Probability Table

wc	wDWS	$P(wc=?, wDWS=?)$
N	H	$\frac{1}{3} * \frac{1}{2} = \frac{1}{6}$
N	T	$\frac{1}{3} * \frac{1}{2} = \frac{1}{6}$
TH	H	$\frac{1}{3} * 1 = \frac{1}{3}$
TH	T	$\frac{1}{3} * 0 = 0$
TL	H	$\frac{1}{3} * 0 = 0$
TL	T	$\frac{1}{3} * 1 = \frac{1}{3}$

$$P(wc=TH | wDWS=H)$$

$$= \frac{P(wc=TH \wedge wDWS=H)}{P(wDWS=H)}$$

$$= \frac{\frac{1}{3}}{\frac{1}{6} + \frac{1}{3} + 0} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$