Nondeterministic Finite Accepters

A *nondeterministic finite accepter* $M$ has a transition function:

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$$

Note that:

- $\delta(q, a)$ can be any subset of $Q$.
- $\delta(q, \lambda)$ allows transitions without reading.

Behavior of NFAs

Start in state $q_0$.

Repeatedly:

Choose $\lambda$ or read the next input symbol.

Choose one of the next states given by $\delta$.

Accept if any possible sequence of choices ends up in a final state.
Examples of NFAs

NFAs and Languages

(not in book) Below are four “set-transition” functions. Let \( R \subseteq Q \).

\( \Lambda(R) \) is the set of states reachable by a \( \lambda \) transition from a state in \( R \).

\[
\Lambda(R) = \bigcup_{q \in R} \delta(q, \lambda)
\]

\( \Lambda^*(R) \) is the set of states reachable by zero or more \( \lambda \) transitions from a state in \( R \).

\[
\Lambda^*(R) = R \cup \Lambda(R) \cup \Lambda(\Lambda(R)) \cup \ldots
\]
Languages Continued

$\Delta(R, a)$ is the set of states reachable by an $a$ transition from a state in $R$.

$$\Delta(R, a) = \bigcup_{q \in R} \delta(q, a)$$

$\Delta^*(R, w)$ is the set of states reachable from a state in $R$ using input string $w$.

$$\Delta^*(R, \lambda) = \Lambda^*(R)$$

$$\Delta^*(R, a) = \Lambda^*(\Delta(\Lambda^*(R), a))$$

$$\Delta^*(R, aw) = \Delta^*(\Delta^*(R, a), w)$$

Languages Continued

The language of an NFA $M$ is defined by:

$$\mathcal{L}(M) = \{w \in \Sigma^* : \emptyset \neq \Delta^*(\{q_0\}, w) \cap F\}$$
Some Properties of NFAs

DFAs and Regular Languages:
DFAs ⊆ NFAs
Regular languages ⊆ languages of NFAs

Closure under Union:
Let $M_1$ and $M_2$ be NFAs.
Some NFA $M$ accepts $L = \mathcal{L}(M_1) \cup \mathcal{L}(M_2)$.
Proof: Copy $M_1$ and $M_2$.
Let $M$’s initial state be a new state with $\lambda$ transitions to the initial states of $M_1$ and $M_2$.

Properties Continued

Closure under Concatenation:
Let $M_1$ and $M_2$ be NFAs.
Some NFA $M$ accepts
\[ L = \{vw : v \in \mathcal{L}(M_1) \land w \in \mathcal{L}(M_2)\} \]
Proof: Copy $M_1$ and $M_2$.
Let $M$’s initial state be $M_1$’s initial state.
Insert $\lambda$ transitions from $M_1$’s final states to $M_2$’s initial state.
Let $M$’s final states be $M_2$’s final states.