Nondeterministic Pushdown Accepters

A *nondeterministic pushdown accepter* $M$ is:

- $Q$, a set of internal states.
- $\Sigma$, the input alphabet. Let $\Sigma_\lambda = (\Sigma \cup \{\lambda\})$.
- $\Gamma$, the stack alphabet.
- $\delta : Q \times \Sigma_\lambda \times \Gamma \rightarrow \text{finite subsets of } Q \times \Gamma^*$.
- $q_0 \in Q$, the initial state.
- $z \in \Gamma$, the start stack symbol.
- $F \subseteq Q$, the final states.

Behavior of NPDAs:

Start in state $q_0$, stack contains $z$

Repeatedly:

- (optional) read the next input symbol,
- pop a symbol off the stack,
- move to the next state given by $\delta$, and
- push symbols on the stack given by $\delta$.

Accept if string read and in final state.
Representations of NPDAs

(not in book) Here are graphical representations of two NPDAs.
Configurations of NPDAs

A running NPDA can be described by \((q, w, u)\) where \(q\) is the current state, \(w\) is the unread part of the input string, and \(u\) is the stack (left top, right bottom).

The first NPDA accepts \(aaabb\).

\[(q_0, aaabb, z) \vdash (q_0, aabb, az) \vdash (q_0, abb, aaz) \vdash (q_1, bb, aaaz) \vdash (q_1, b, aaz) \vdash (q_1, \lambda, az)\]

What are examples of strings that are rejected?

The second NPDA accepts \(bbaaab\).

\[(q_0, bbaaab, z) \vdash (q_2, baaaab, bz) \vdash (q_2, aab, bbz) \vdash (q_2, aab, bz) \vdash (q_2, ab, z) \vdash (q_0, ab, z) \vdash (q_1, b, az) \vdash (q_1, \lambda, z) \vdash (q_0, \lambda, z)\]

What are examples of strings that are rejected?