Unrestricted Grammars

An *unrestricted grammar* has productions:

$$u \longrightarrow v$$

where $u \in (V \cup T)^+$ and $v \in (V \cup T)^*$.
That is, productions with no restrictions.

UG = recursively enumerable = Turing machines
A TM can simulate a UG.
An UG can simulate TM configurations.

Context-Sensitive Grammars

A *context-sensitive grammar* has productions:

$$x \longrightarrow y$$

where $x, y \in (V \cup T)^+$ and $|x| \leq |y|$.
That is, derivations are *noncontracting*.

This grammar accepts $\{a^n b^n c^n : n \geq 1\}$.

$$\begin{aligned} S &\longrightarrow abc \\ ab &\longrightarrow aabbC \\ Cb &\longrightarrow bC \\ Cc &\longrightarrow cc \end{aligned}$$

Properties of Context-Sensitive Languages

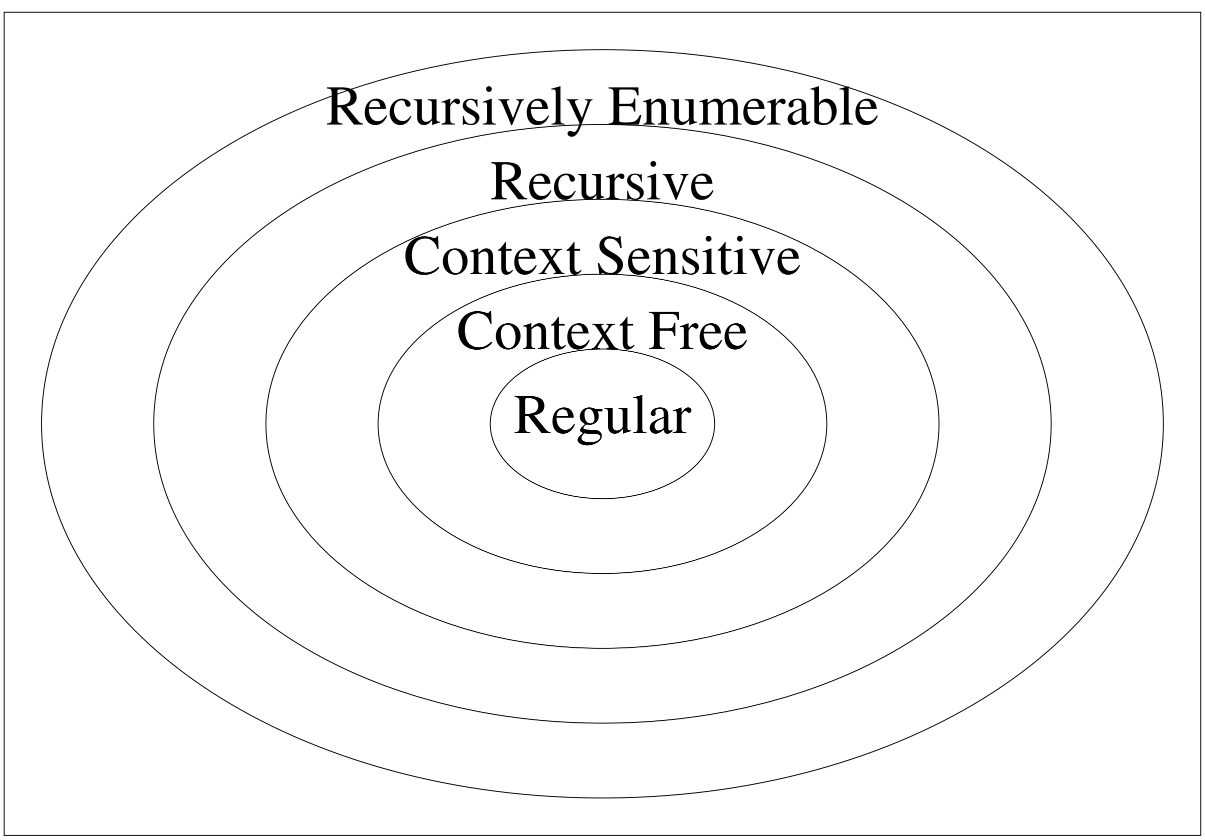
CFLs $\subseteq$ CSLs. Convert CFG to Chomsky normal form, which is a CSG.

CFLs $\neq$ CSLs. No CFG for $\{a^n b^n c^n : n \geq 1\}$.

CSLs $\subseteq$ recursive languages.
Derivation of w cannot be longer than m^{n+1}, where $m = |V \cup T|$ and $n = |w|$.

CSLs $\neq$ recursive languages. Enumerate strings $x_1, x_2, x_3, \ldots$ and CSGs $G_1, G_2, G_3, \ldots$.
Let $G' = \{x_i : x_i \notin \mathcal{L}(G_i)\}$. Diagonalized!

Hierarchy of Languages



An Example CSG

This grammar accepts $\{ab^{n^2}c : n \geq 1\}$.

$$
\begin{aligned}
S &\rightarrow aDc & bZ &\rightarrow Zb \\
D &\rightarrow Dd \mid X & cZ &\rightarrow Zc \\
Xb &\rightarrow bX & dZ &\rightarrow Zd \\
Xc &\rightarrow bc \\
Xd &\rightarrow bbbY \\
Yb &\rightarrow bY \\
Yc &\rightarrow Zc \\
Yd &\rightarrow bbdY
\end{aligned}
$$