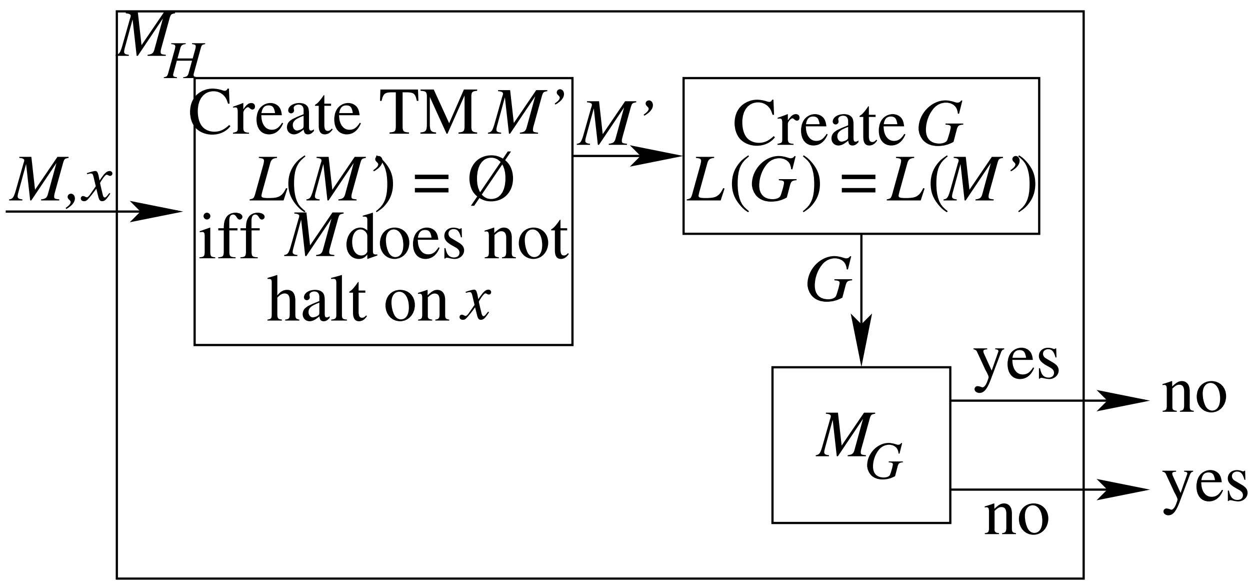
Undecidable Problems

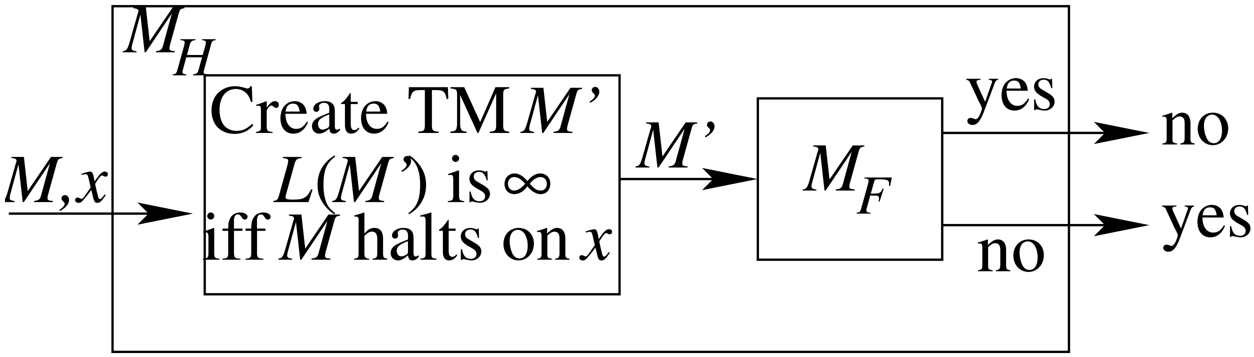
Problem: For unrestricted grammars, is $\mathcal{L}(G) = \emptyset$?

Reduction: Map M, x to a TM M' that rejects all inputs except that it accepts x if M halts on x.

Map this TM to an unrestricted grammar (Theorem 11.7).



Problem: Is $\mathcal{L}(M)$ a finite language?
Reduction: Map M, x to a TM M' that accepts all inputs if M halts on x, else it accepts $\emptyset$.



Rice's Theorem

Let P be any decision problem on TMs that satisfies:

1. If $\mathcal{L}(M_1) = \mathcal{L}(M_2)$, then the decision problem gives the same answer to M_1 and M_2.
2. There exist M_1 and M_2 such that the decision problem gives different answers.

It follows that P is undecidable.

The Post Correspondence Problem

Let $A = w_1, w_2, \ldots, w_n$ and $B = v_1, v_2, \ldots, v_n$ be two finite sequences of strings. For example,

$$A = i, mi, p, pi, ss \qquad B = s, m, ipp, i, is$$

A solution to the Post Correspondence Problem is a finite sequence of integers $i_1, i_2, \ldots, i_k$ such that:

$$w_{i_1} w_{i_2} \cdots w_{i_k} = v_{i_1} v_{i_2} \cdots v_{i_k}$$

This problem is undecidable.

More Undecidable Problems

Is a Turing machine the smallest TM for computing its function?

Is a context-free language ambiguous?

Is the language of a context-free grammar equal to Σ^*?

Is the intersection of two context-free languages equal to $\emptyset$?

Is a proposition using natural numbers, addition, and multiplication true?