

P and **NP**

P is the set of problems solvable in polynomial time.

$$\mathbf{P} = \bigcup_{i \geq 1} \mathit{DTIME}(n^i)$$

NP is the set of problems solvable in nondeterministic polynomial time.

$$\mathbf{NP} = \bigcup_{i \geq 1} \mathit{NTIME}(n^i)$$

Obviously, $\mathbf{P} \subseteq \mathbf{NP}$. It is generally believed that $\mathbf{P} \neq \mathbf{NP}$, but no proof is known.

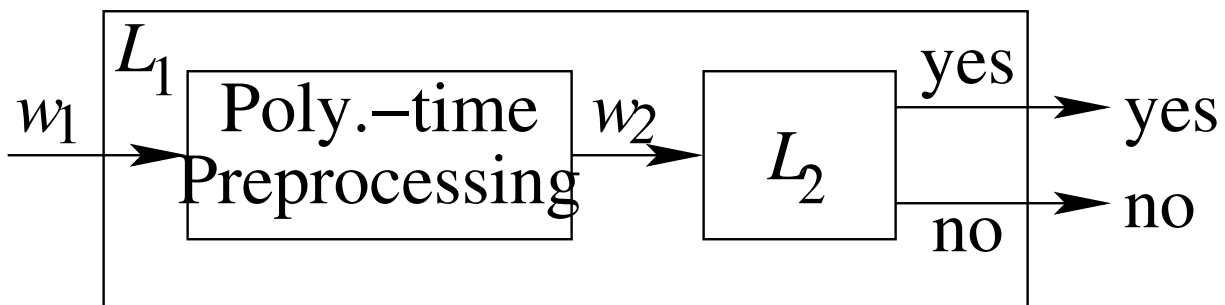
A crude distinction is to call a problem *tractable* if it is in **P**; otherwise, the problem is *intractable*.

In reality, only low-order polynomial time is tractable, e.g., $\mathit{DTIME}(n^3)$

Some problems not in **P** appear tractable on average.

Reductions and **NP**-Complete Problems

A language L_1 is *polynomial-time reducible* to L_2 if a poly-time algorithm can convert any w_1 to w_2 such that $w_1 \in L_1$ iff $w_2 \in L_2$.



A language L is **NP-complete** if $L \in NP$ and if every $L' \in \mathbf{NP}$ is poly-time reducible to L .

SAT is **NP**-complete.

Input: a Boolean expression

Output: Answer yes if it is satisfiable, i.e., if any value assignment makes it true.

In **NP**: Nondeterministically choose values, and check for satisfiability.

Is **NP**-complete: Difficult proof.

Some **NP**-Complete Problems

3SAT is a Boolean problem.

Input: A conjunction $t_1 \wedge t_2 \wedge \dots \wedge t_k$.

Each c_i is a disjunction $l_{i,1} \vee l_{i,2} \vee l_{i,3}$.

Each $l_{i,j}$ is a Boolean variable or its negation.

Boolean variables are reusable.

Output: Answer yes if it is satisfiable.

In **NP**: Nondeterministically choose values, and check for satisfiability.

Is **NP**-complete: Reduce SAT to 3SAT

CLIQUE is a graph problem.

Input: An undirected graph and a pos. int. k

Output: Answer yes if it has a clique of size k , i.e., there is a subset of k vertices that are completely connected to each other.

In **NP**: Nondeterministically choose k vertices, and check if they form a clique.

Is **NP**-complete: Reduce 3SAT to CLIQUE

An Example Reduction

A polynomial time reduction from 3SAT to CLIQUE has the following steps.

1. Map each conjunct to three vertices.
2. Add an edge between every pair of vertices except when they are in the same conjunct or they correspond to a variable and its negation.

There is a satisfying assignment iff there is a clique of size k .

