Nondeterministic Finite Accepters

A *nondeterministic finite accepter* M has a transition function:

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$$

Note that:

$\delta(q, a)$ can be any subset of Q.

$\delta(q, \lambda)$ allows transitions without reading.

Behavior of NFAs

Start in state q_0.

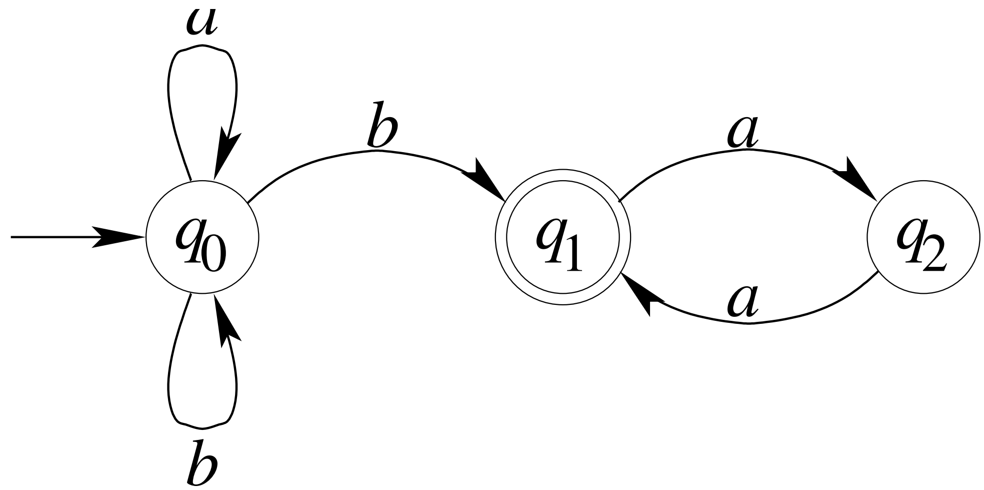
Repeatedly:

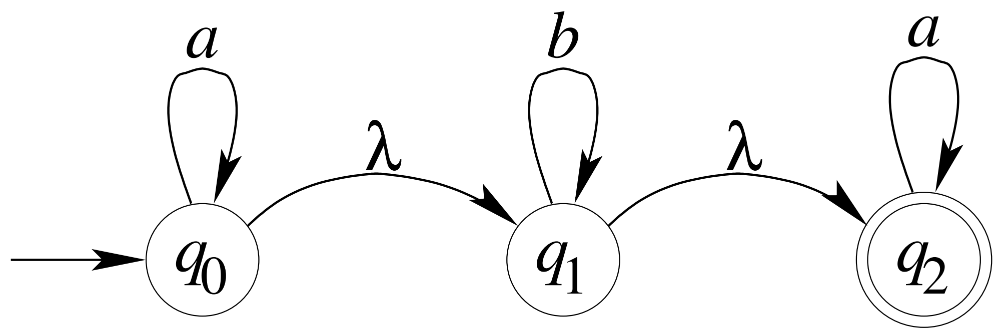
Choose λ or read the next input symbol.

Choose one of the next states given by δ.

Accept if any possible sequence of choices ends up in a final state.

Examples of NFAs





NFAs and Languages

(not in book) Below are four "set-transition" functions. Let $R \subseteq Q$.

$\Lambda(R)$ is the set of states reachable by a λ transition from a state in R.

$$\Lambda(R) = \bigcup_{q \in R} \delta(q, \lambda)$$

$\Lambda^*(R)$ is the set of states reachable by zero or more λ transitions from a state in R.

$$\Lambda^*(R) = R \cup \Lambda(R) \cup \Lambda(\Lambda(R)) \cup \ldots$$

Languages Continued

$\Delta(R, a)$ is the set of states reachable by an a transition from a state in R.

$$\Delta(R, a) = \bigcup_{q \in R} \delta(q, a)$$

$\Delta^*(R, w)$ is the set of states reachable from a state in R using input string w.

$$\Delta^*(R, \lambda) = \Lambda^*(R)$$

$$\Delta^*(R, a) = \Lambda^*(\Delta(\Lambda^*(R), a))$$

$$\Delta^*(R, aw) = \Delta^*(\Delta^*(R, a), w)$$

Languages Continued

The language of an NFA M is defined by:

$$\mathcal{L}(M) = \{w \in \Sigma^* : \emptyset \neq \Delta^*(\{q_0\}, w) \cap F\}$$

Some Properties of NFAs

DFAs and Regular Languages:
DFAs $\subseteq$ NFAs
Regular languages $\subseteq$ languages of NFAs

Closure under Union:
Let M_1 and M_2 be NFAs.
Some NFA M accepts $L = \mathcal{L}(M_1) \cup \mathcal{L}(M_2)$.
Proof: Copy M_1 and M_2.
Let M's initial state be a new state with λ transitions to the initial states of M_1 and M_2.

Properties Continued

Closure under Concatentation:
Let M_1 and M_2 be NFAs.
Some NFA M accepts
$L = \{vw : v \in \mathcal{L}(M_1) \wedge w \in \mathcal{L}(M_2)\}$
Proof: Copy M_1 and M_2.
Let M's initial state be M_1's initial state.
Insert λ transitions from M_1's final states to M_2's initial state.
Let M's final states be M_2's final states.