

Context-Free Grammars

A *context-free grammar* has productions:

$$A \rightarrow x$$

where $A \in V$ and $x \in (V \cup T)^*$

Example 1: Balanced Parentheses

$$S \rightarrow SS$$

$$S \rightarrow (S)$$

$$S \rightarrow \lambda$$

Example 2: Arithmetic Expressions (start E)

$$E \rightarrow V$$

$$E \rightarrow EOE$$

$$E \rightarrow (E)$$

$$V \rightarrow a \mid b \mid c$$

$$O \rightarrow + \mid - \mid * \mid /$$

Derivations and Trees

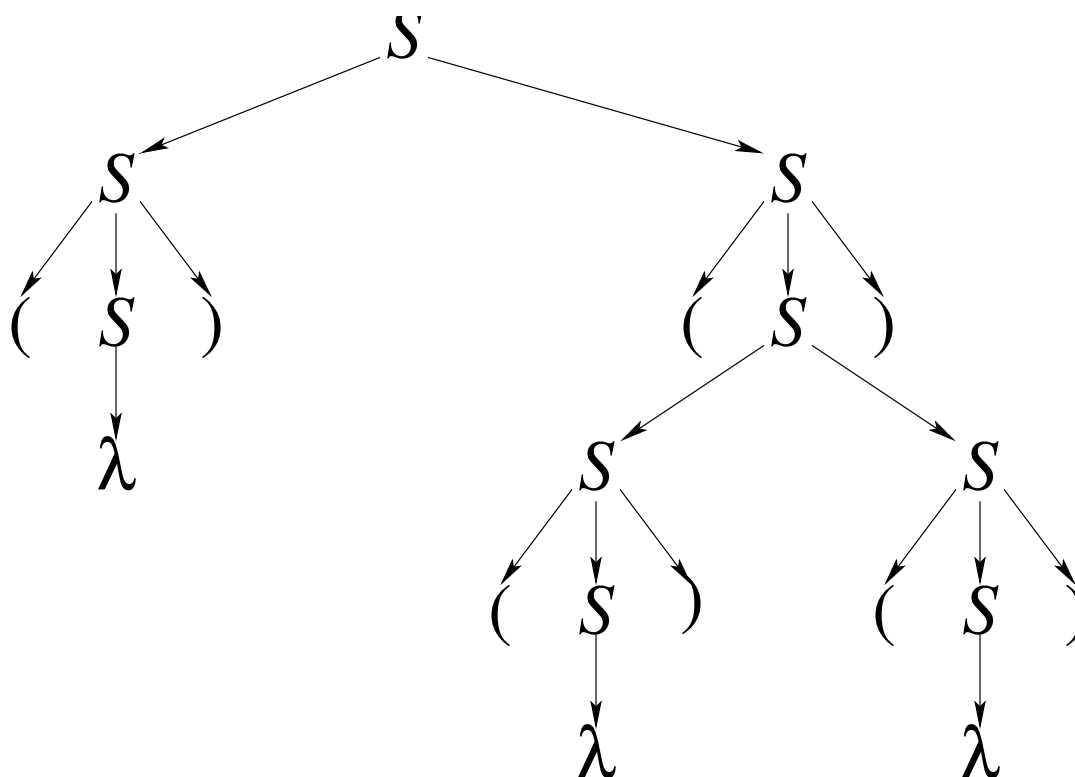
Leftmost Derivation of $()(())()$

$$\begin{aligned} S &\Rightarrow SS \Rightarrow (S)S \Rightarrow ()S \Rightarrow ()(S) \Rightarrow ()(SS) \\ &\Rightarrow ()((S)S) \Rightarrow ()(())S \Rightarrow ()(())(S) \Rightarrow ()(())() \end{aligned}$$

Rightmost Derivation of $()(())()$

$$\begin{aligned} S &\Rightarrow SS \Rightarrow S(S) \Rightarrow S(SS) \Rightarrow S(S(S)) \\ &\Rightarrow S(S()) \Rightarrow S((S)()) \Rightarrow S(()()) \Rightarrow \\ &(S)()() \Rightarrow ()(())() \end{aligned}$$

Derivation Tree for $()(())()$



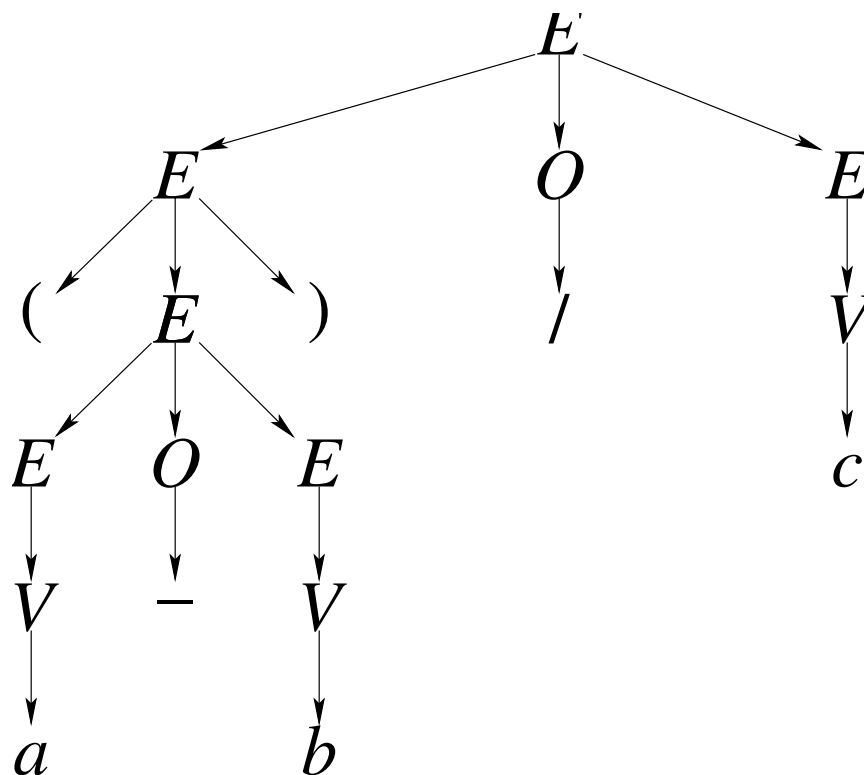
Leftmost Derivation of $(a - b)/c$

$$\begin{aligned}
 E &\Rightarrow EOE \Rightarrow (E)OE \Rightarrow (EOE)OE \Rightarrow \\
 &(VOE)OE \Rightarrow (aOE)OE \Rightarrow (a - E)OE \\
 &\Rightarrow (a - V)OE \Rightarrow (a - b)OE \Rightarrow \\
 &(a - b)/E \Rightarrow (a - b)/V \Rightarrow (a - b)/c
 \end{aligned}$$

Rightmost Derivation of $(a - b)/c$

$$\begin{aligned}
 E &\Rightarrow EOE \Rightarrow EOV \Rightarrow EOc \Rightarrow E/c \Rightarrow \\
 &(E)/c \Rightarrow (EOE)/c \Rightarrow (EOV)/c \Rightarrow (EOb)/c \\
 &\Rightarrow (E - b)/c \Rightarrow (V - b)/c \Rightarrow (a - b)/c
 \end{aligned}$$

Derivation Tree for $(a - b)/c$



BNF (Backus-Naur Form) Grammars

BNF is a different notation for CFGs,
traditionally used for programming languages.

Example 1: Balanced Parentheses

```
<parens> ::= <parens> <parens>
<parens> ::= ( <parens> )
<parens> ::= ""
```

Example 2: Arithmetic Expressions (start E)

```
<exp> ::= <var>
<exp> ::= <exp> <op> <exp>
<exp> ::= ( <exp> )
<var> ::= a | b | c
<op> ::= + | - | * | /
```