Turing Machines

A *Turing machine* M is:

Q, a set of internal states.

Σ, the input alphabet.

Γ, tape alphabet, $\Sigma \subset \Gamma$, $\square \in \Gamma - \Sigma$.

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

$q_0 \in Q$, the initial state.

$F \subseteq Q$, the final states.

Behavior of Turing machine M on input w

Initial state q_0, tape w, blanks around w, read-write head at first symbol.

Repeatedly:

Read-write head reads tape symbol.

Halt if no δ for current state, symbol.

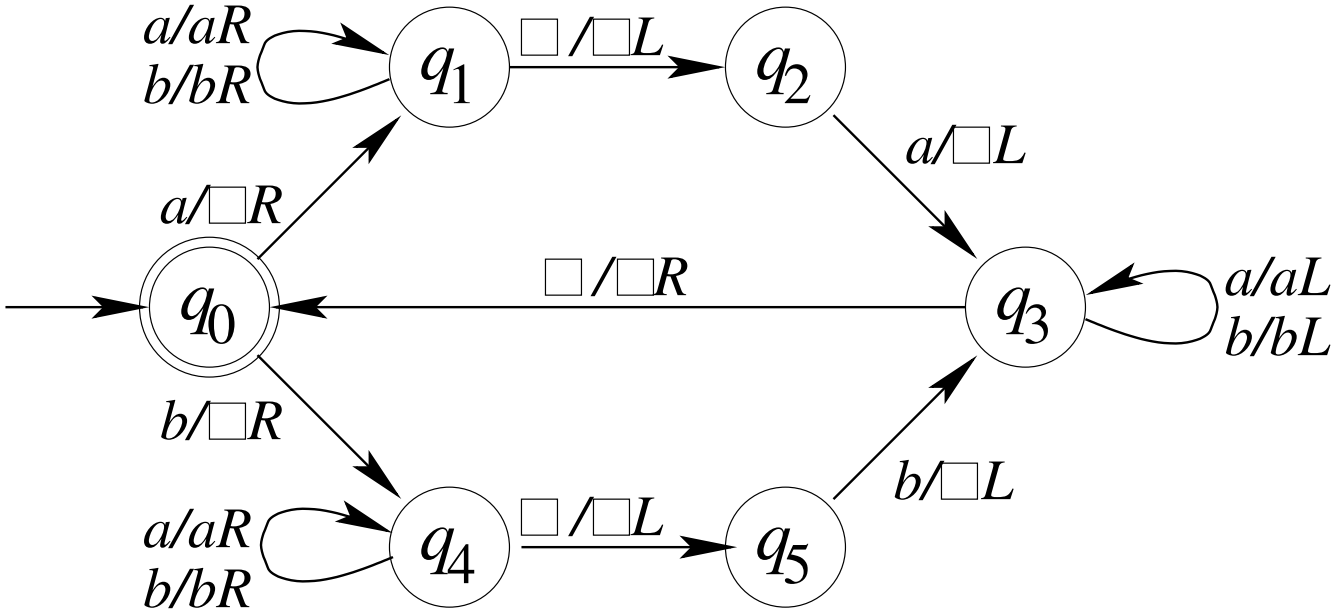
Read-write head writes tape symbol.

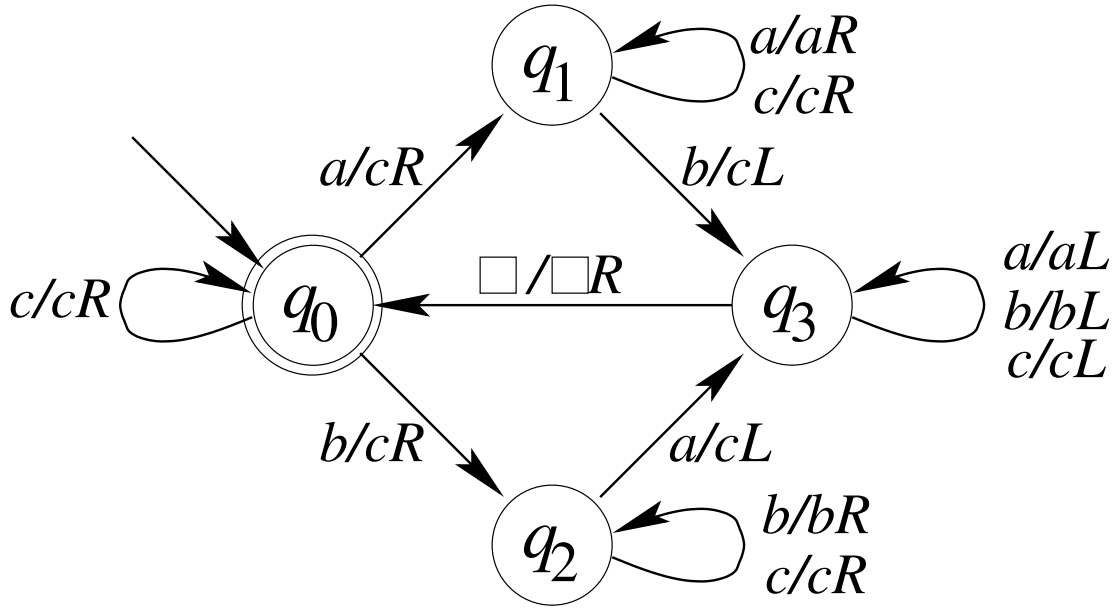
Read-write head moves left or right.

New state given by δ

Accept if M halts in a final state.

(not in book) Here are transition graphs for two TMs.





Configurations of TMs

A running TM can be described by $xqay$ where q is the current state, the read-write head points at a, x is to the left of the read-write head, and y is to the right of the read-write head.

The first TM accepts $baab$.

$q_0baab \vdash \square q_4aab \overset{*}{\vdash} \square aabq_4 \vdash \square aaq_5b \vdash$
$\square aq_3a\square \overset{*}{\vdash} q_3\square aa\square \vdash \square q_0aa\square \vdash \square\square q_1a\square \vdash$
$\square\square aq_1\square \vdash \square\square q_2a\square \vdash \square q_3\square\square\square \vdash \square\square q_0\square\square$

The second TM accepts $bbaa$.

$q_0bbaa \vdash cq_2baa \vdash cbq_2aa \vdash cq_3bca \vdash q_3cbca \vdash$
$q_3\square cbca \vdash q_0cbca \vdash cq_0bca \vdash ccq_2ca \vdash$
$cccq_2a \vdash ccq_3cc \overset{*}{\vdash} q_3\square cccc \vdash q_0cccc \overset{*}{\vdash} ccccq_0$

Computing with TMs

We can also use TMs to perform computations.
A function f is *Turing-computable* iff
there exists some TM M such that

$f(x) = y$
iff there exists some final state q_f such that
$q_0 x \overset{*}{\vdash} q_f y$ and halts.

Next is a TM that computes $f(x) = 2x$, where x is a positive integer represented in unary notation, e.g., 5 is represented as 11111.