SMO Algorithm

Initialize $\alpha_i \leftarrow 0$ and $b \leftarrow 0$

Let $f_a(x) = b + \sum_{i=1}^{m} y_i \alpha_i k(x, x_i)$

Let $\tau$ be the tolerance.

Loop

Find two exs. $(x_p, y_p)$ and $(x_q, y_q)$ such that:

$$(f_a(x_p) - y_p + \tau < f_a(x_q) - y_q - \tau) \land$$

$$((\alpha_p < C \land y_p = 1) \lor (\alpha_p > 0 \land y_p = -1)) \land$$

$$((\alpha_q > 0 \land y_q = 1) \lor (\alpha_q < C \land y_q = -1))$$

exit loop if no such examples can be found

$$\eta \leftarrow \frac{(f_a(x_q) - y_q) - (f_a(x_p) - y_p)}{k(x_p, x_p) - 2k(x_p, x_q) + k(x_q, x_q)}$$

if needed, reduce $\eta$ so that:

$$(0 \leq \alpha_p + y_p \eta \leq C) \land$$

$$(0 \leq \alpha_q - y_q \eta \leq C)$$

$$\alpha_p \leftarrow \alpha_p + y_p \eta$$

$$\alpha_q \leftarrow \alpha_q - y_q \eta$$

End Loop

$b_1 \leftarrow \max \{ f_a(x_i) \mid y_i = 1 \land \alpha_i > 0 \}$

$b_{-1} \leftarrow \min \{ f_a(x_i) \mid y_i = -1 \land \alpha_i > 0 \}$

$b \leftarrow -(b_1 + b_{-1})/2$
SMO Explanation

SMO means “sequential minimal optimization”.

The conditions on $x_p$ and $x_q$ mean:

1. Relative to each other, $x_p$ is below its margin boundary, and $x_q$ is above its boundary.
2. The tolerance is a tradeoff between accuracy and efficiency.
3. $\alpha_p$ can be changed to increase $f_a(x_p)$.
4. $\alpha_q$ can be changed to decrease $f_a(x_q)$.

To maintain $\sum_{i=1}^{m} \alpha_i y_i = 0$, an increase in $\alpha_p y_p$ is matched by a decrease in $\alpha_q y_q$.

$\eta$ optimizes the objective function.

$0 \leq \alpha_i \leq C$ must be maintained.

One way to set the bias weight $b$ is shown.

Many details to make this more efficient and stable have been omitted.