

Using Examples for Classification

A Simple Algorithm

Suppose we model a class by its average example and predict the class of \mathbf{x} by testing which average is closest.

$$\mathbf{c}_+ = \sum_{y_i=1} \mathbf{x}_i / m_+ \quad \mathbf{c}_- = \sum_{y_i=-1} \mathbf{x}_i / m_-$$

m_+ and m_- are respectively the number of positive and negative examples.

The class of \mathbf{x} is predicted by:

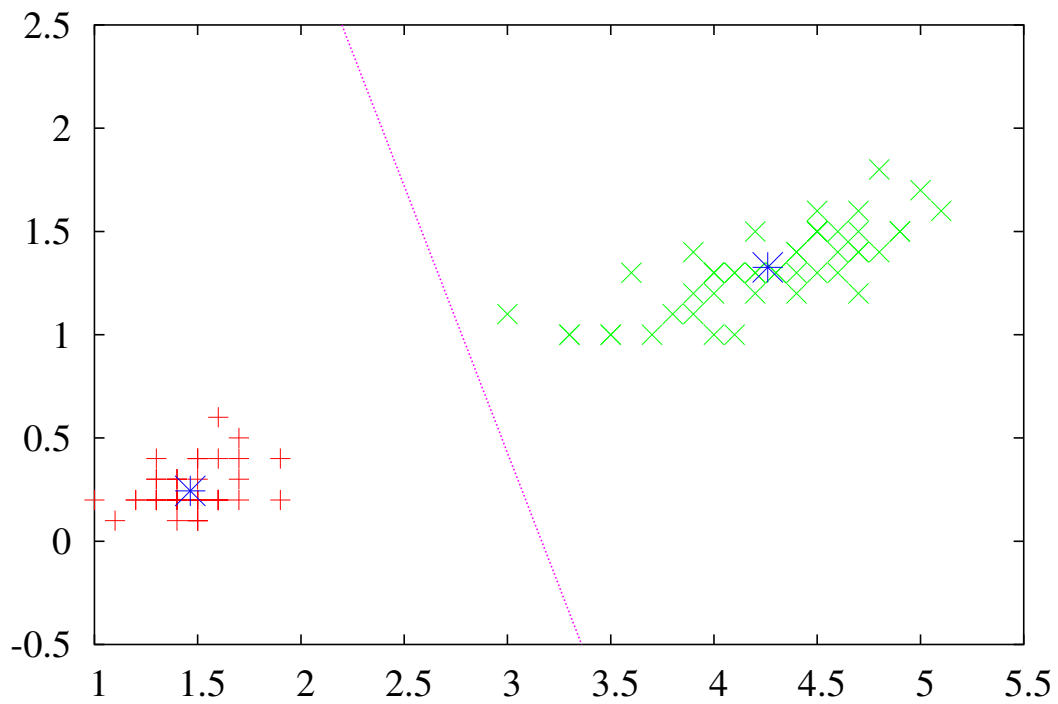
$$\text{sign} \left(\|\mathbf{c}_- - \mathbf{x}\|^2 - \|\mathbf{c}_+ - \mathbf{x}\|^2 \right)$$

$$\begin{aligned} & \text{sign} \left(\|\mathbf{c}_- - \mathbf{x}\|^2 - \|\mathbf{c}_+ - \mathbf{x}\|^2 \right) \\ &= \text{sign} \left(2(\mathbf{x} \cdot \mathbf{c}_+) - 2(\mathbf{x} \cdot \mathbf{c}_-) \right. \\ & \quad \left. + (\mathbf{c}_- \cdot \mathbf{c}_-) - (\mathbf{c}_+ \cdot \mathbf{c}_+) \right) \\ &= \text{sign} \left(\frac{2 \sum_{y_i=1} (\mathbf{x} \cdot \mathbf{x}_i)}{m_+} - \frac{2 \sum_{y_i=-1} (\mathbf{x} \cdot \mathbf{x}_i)}{m_-} + b \right) \end{aligned}$$

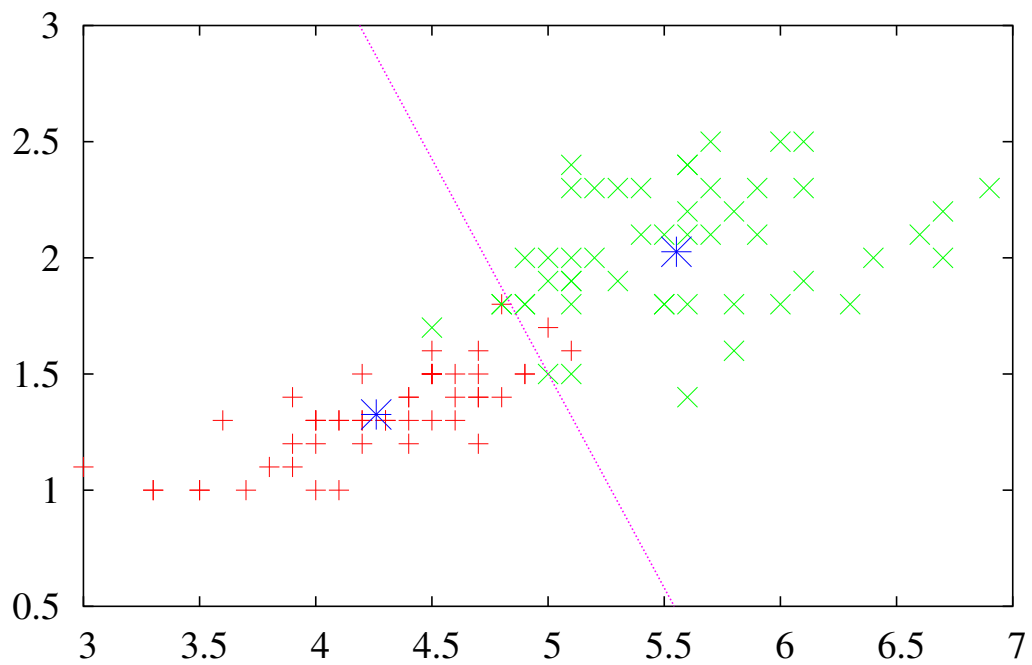
where $b = (\mathbf{c}_- \cdot \mathbf{c}_-) - (\mathbf{c}_+ \cdot \mathbf{c}_+)$. Like a SVM, the decision function has the form:

$$\text{sign} \left(b + \sum_i \alpha_i y_i k(\mathbf{x} \cdot \mathbf{x}_i) \right)$$

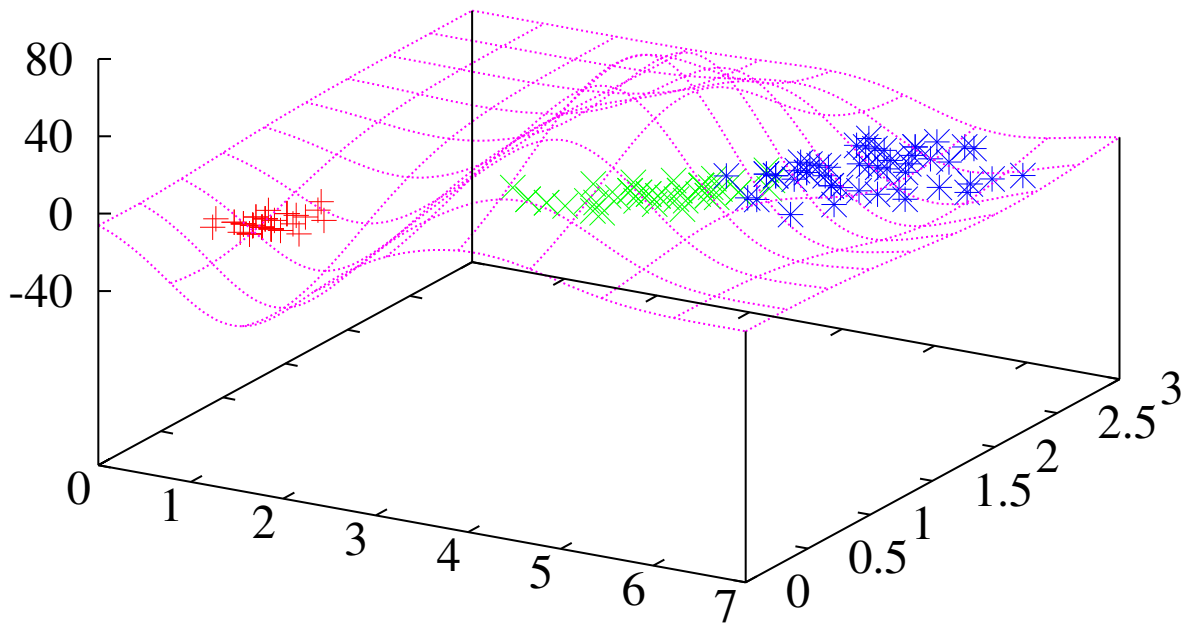
Example 1 of Simple Algorithm



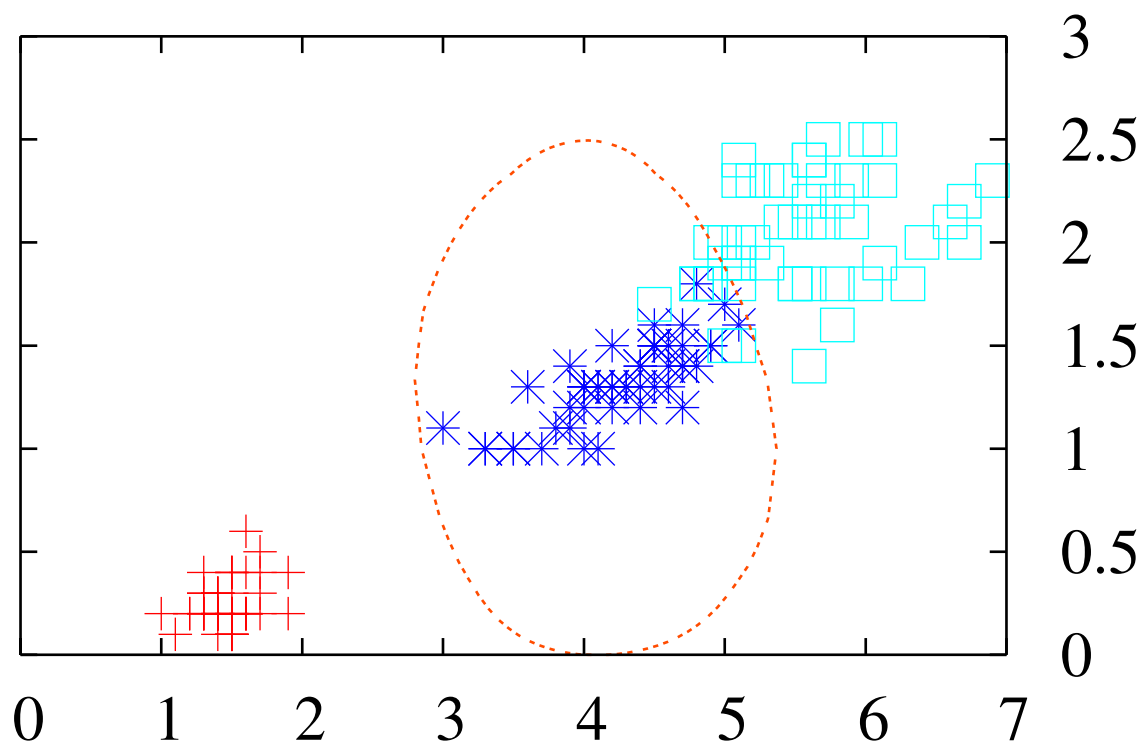
Example 2 of Simple Algorithm



Example 3 Using Gaussian Kernel



Example 3 Decision Boundary



Comparison to Properties of SVMs

SVMs also represent decision boundaries as a linear combination of examples.

SVMs weights (α_i) will range between 0 and a specified constant C .

SVMs use zero weights for correctly classified examples “far away” from the decision boundary.