Decidable Languages

Our power is in our ability to decide.
(Richard Buckminster Fuller)

Languages Based On Automata

Each automata we have studied correspond to a class of languages and can be represented by strings.

- Regular languages: regular expressions.
- Context-free languages: CFGs.
- Turing-recogizable languages: TMs.

Problems about automata can be transformed into languages.

- Determine if two CFLs are equal.
- Determine if two CFGs have same langauge.
- $\{x\#y \mid x \text{ and } y \text{ are CFGs} \wedge L(x) = L(y)\}$

Problems Concerning Regular Languages

$A_{\text{DFA}} = \{\langle B, w\rangle \mid B \text{ is a DFA and } w \in L(B)\}$
A_{DFA} is a decidable language.
Proof Sketch: It's easy to simulate B.

$A_{\text{NFA}} = \{\langle B, w\rangle \mid B \text{ is a NFA and } w \in L(B)\}$
A_{NFA} is a decidable language.
Proof Sketch: Convert B to a DFA C and test if $\langle C, w\rangle$ is in A_{DFA}.

$A_{\text{REX}} = \{\langle R, w\rangle \mid R \text{ is a regex. and } w \in L(R)\}$
A_{REX} is a decidable language.
Proof Sketch: Convert R to a DFA B and test if $\langle B, w\rangle$ is in A_{DFA}.

More Regular Language Problems

$E_{\text{DFA}} = \{\langle B \rangle \mid B \text{ is a DFA and } L(B) = \emptyset\}$
E_{DFA} is a decidable language.
Proof Sketch: Determine if an accept state is reachable from the start state.

$EQ_{\text{DFA}} =$
$\{\langle B, C \rangle \mid B \text{ and } C \text{ are DFAs and } L(B) = L(C)\}$
EQ_{DFA} is a decidable language.
Proof Sketch: Construct a DFA D for
$(B \cap \overline{C}) \cup (\overline{B} \cap C)$ and test if $\langle D, w \rangle$ is in E_{DFA}.

Problems Concerning Context-Free Langs.

$A_{\text{CFG}} = \{\langle G, w \rangle \mid G \text{ is a CFG and } w \in L(G)\}$
A_{CFG} is a decidable language.
Proof Sketch: Incrementally construct a table of what variables generate what substrings.

$E_{\text{CFG}} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$
E_{CFG} is a decidable language.
Proof Sketch: Incrementally mark variables that can generate a sequence of terminals.

Every CFG is decidable.
Proof Sketch: Test if $\langle G, w \rangle$ is in A_{CFG}.

Venn Diagram of Classes of Languages