1. Show $\lg(n!)$ is $\Theta(n \lg n)$. Note that you need to show both $O(n \lg n)$ and $\Omega(n \lg n)$.
2. Use a recursion tree to show that $T(n) = T(9n/10) + T(n/10)$ is $\Theta(n \lg n)$.
3. Use the Master Theorem to analyze $T(n) = 3T(n/2) + n$.
4. Show a trace of the Partition algorithm on a given array.
5. Show a trace of the Max-Heapify algorithm on a given array.
6. Insert a sequence of values into a red-black tree.
7. Insert a sequence of values into a hash table using linear probing for collisions.
8. Write the Select algorithm for finding the $i$th order statistic using the Partition algorithm.
9. Justify that the Select algorithm is $\Theta(n)$ on average.
10. For the following procedure:

\[
\text{Random-Max}(n) \\
\begin{align*}
x &\leftarrow 1 \\
\text{for } i &\leftarrow 1 \text{ to } n \\
& \quad r \leftarrow \text{RANDOM}(1, 6) \\
& \quad x \leftarrow \max(x, r) \\
\text{return } x
\end{align*}
\]

(a) What is the probability that \text{Random-Max}(1) returns 6?
(b) What is the probability that \text{Random-Max}(2) returns 6?
(c) What is the probability that \text{Random-Max}(13) returns 6?