

LESSON 9: Logarithmic scales and growth rates

FOCUS QUESTION: How can I use logarithmic scales to understand rates of growth?

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EXAMPLE 1: Load the fetal size data

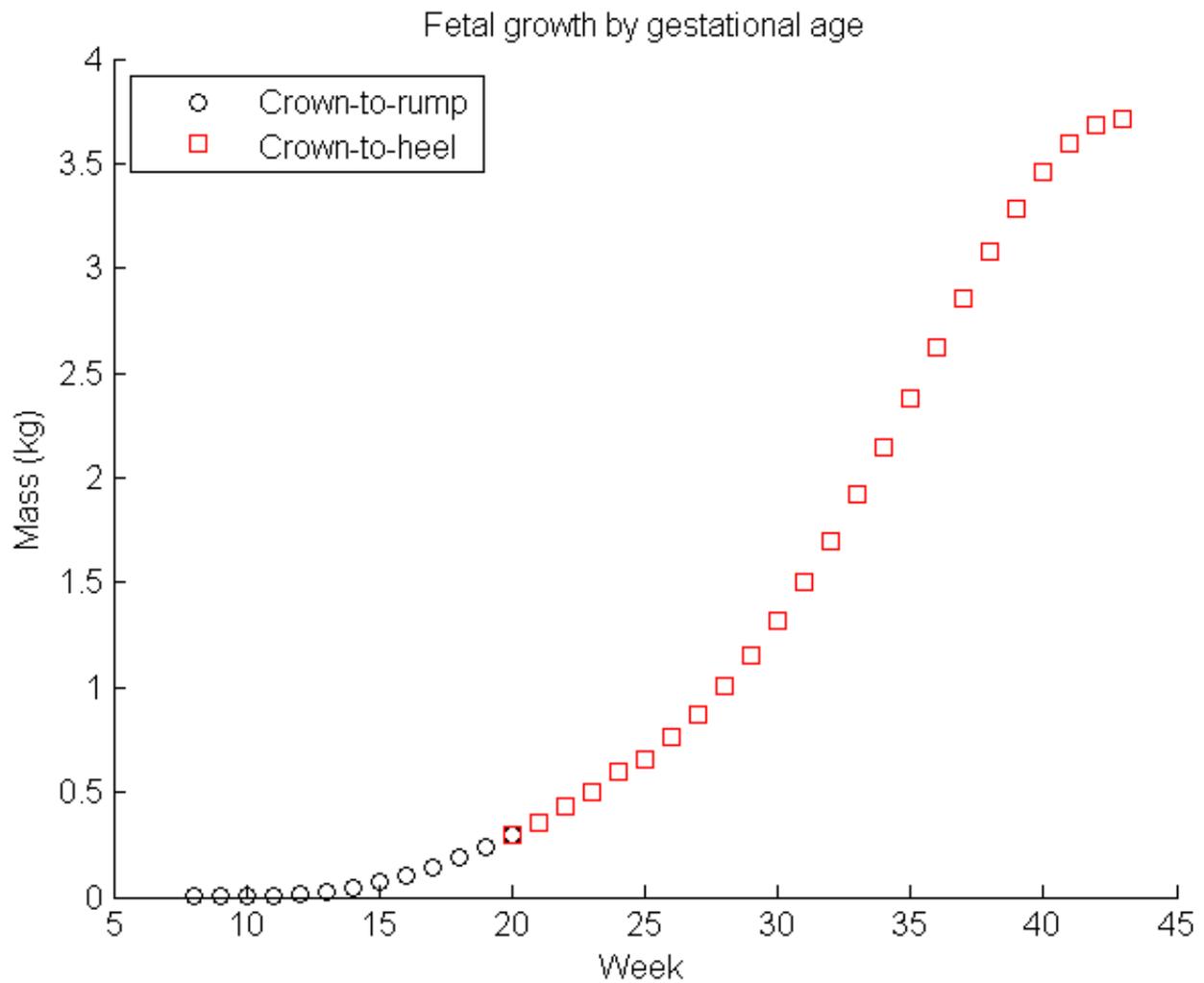
```
load toRump.txt; % Load the crown-to-rump data
load toHeel.txt; % Load the crown-to-heel data
```

EXAMPLE 2: Define variables to make code easier to read

```
rWeeks = toRump(:, 1); % Weeks in the toRump measurements
hWeeks = toHeel(:, 1); % Weeks in the toHeel measurements
rMass = toRump(:, 3)/1000; % Mass in kg of toRump measurements
hMass = toHeel(:, 3)/1000; % Mass in kg of toHeel measurements
```

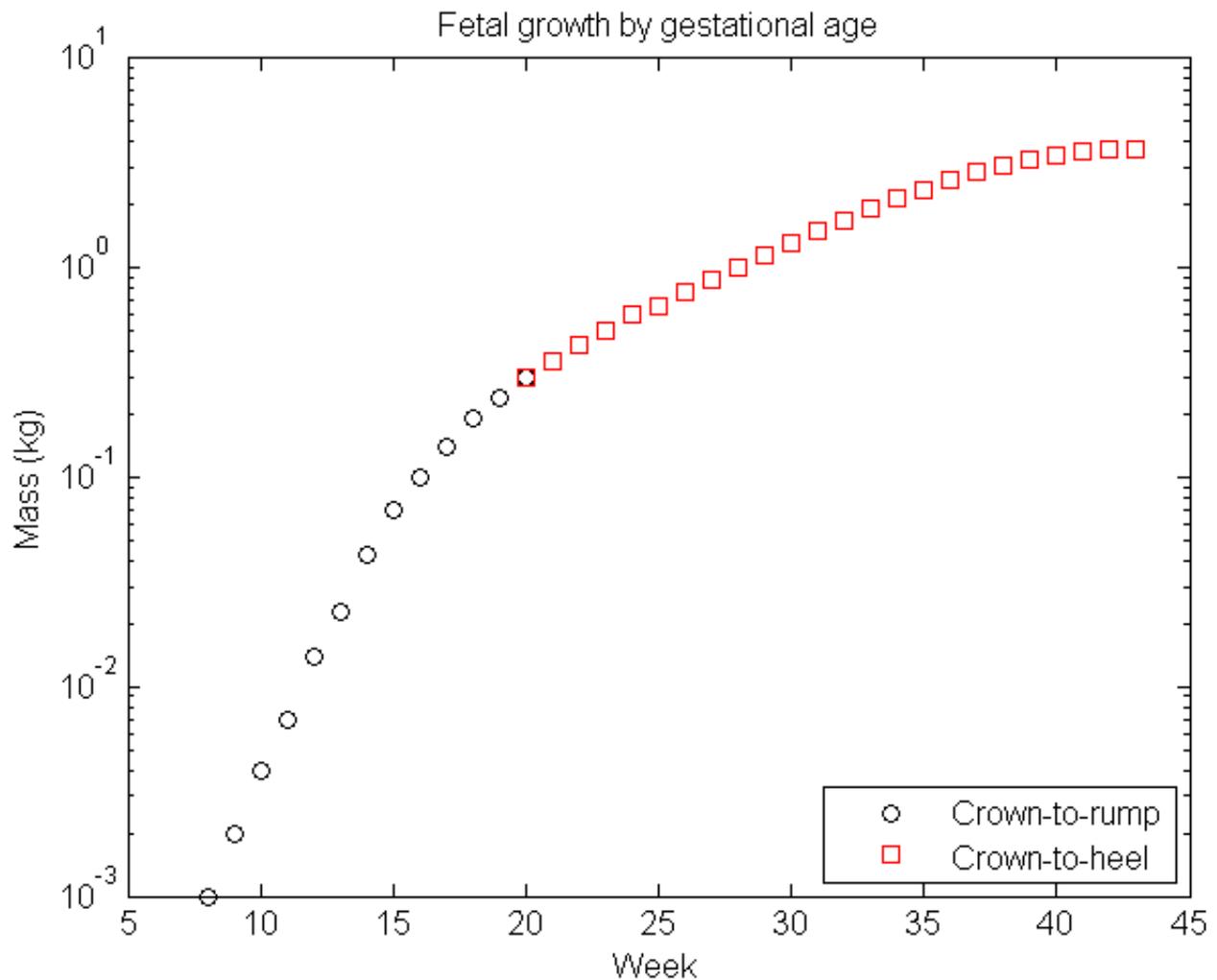
EXAMPLE 3: Plot fetal mass as a function of gestational week

```
figure
hold on
plot(rWeeks, rMass, 'ko') % Plot the toRump mass
plot(hWeeks, hMass, 'rs') % Plot the toHeel mass
xlabel('Week');
ylabel('Mass (kg)');
title('Fetal growth by gestational age');
legend({'Crown-to-rump', 'Crown-to-heel'}, 'Location', 'NorthWest')
hold off%%
```



EXAMPLE 4: Plot fetal mass as a function of gestational week using a semilog scale

```
figure
semilogy(rWeeks, rMass, 'ko', hWeeks, hMass, 'rs') % Plot the toRump mass
xlabel('Week');
ylabel('Mass (kg)');
title('Fetal growth by gestational age');
legend({'Crown-to-rump', 'Crown-to-heel'}, 'Location', 'SouthEast')
```



Questions	Answers
What does <code>semilogy(x, y)</code> do?	The <code>semilogy</code> function plots x versus the $\log_{10}(y)$.
Why is the y-axis labeled in kg rather than the log of kg?	The <code>semilogy</code> function, chooses major tick marks that correspond to powers of 10 in the original data. The ticks marks are labeled with the values in the original data. Hence, the reader uses original units.
Why not use two calls to <code>semilogy</code> with <code>hold on</code> ?	The <code>semilogy</code> function does not work with <code>hold on</code> . The plot reverts to linear scale.

EXAMPLE 5: Estimate fetal mass growth rates using the slope from the linear fit

```
pRump = polyfit(rWeeks, log10(rMass), 1); % Fit weeks 8-20
pHeel = polyfit(hWeeks, log10(hMass), 1); % Fit weeks 20-43
corRump = corr(rWeeks, log10(rMass)); % Find correlation
corHeel = corr(hWeeks, log10(hMass));
```

EXAMPLE 6: Output the estimates of fetal mass growth rates and R^2 for the linear fits

```
fprintf(['Fetal mass growth rates in g/week:\n' ...
```

```
\t%g g/weeks 8-20, R^2 = %g\n' ...
\t%g g/weeks 20-43, R^2 = %g\n'], ...
pRump(1), corRump*corRump, pHeel(1), corHeel*corHeel);
```

Fetal mass growth rates in g/week:

```
0.209119 g/weeks 8-20, R^2 = 0.971853
0.0491613 g/weeks 20-43, R^2 = 0.972093
```

Questions	Answers
What is R2?	The R2 is a measure of how much of the data variance is accounted for by the model (after the mean is taken out). A value of R2 close to one indicates a good model.
Is R2 always the square of the correlation?	In general, no. However, for linear models (i.e., fitting a straight line), the value of R2 is the square of the correlation.

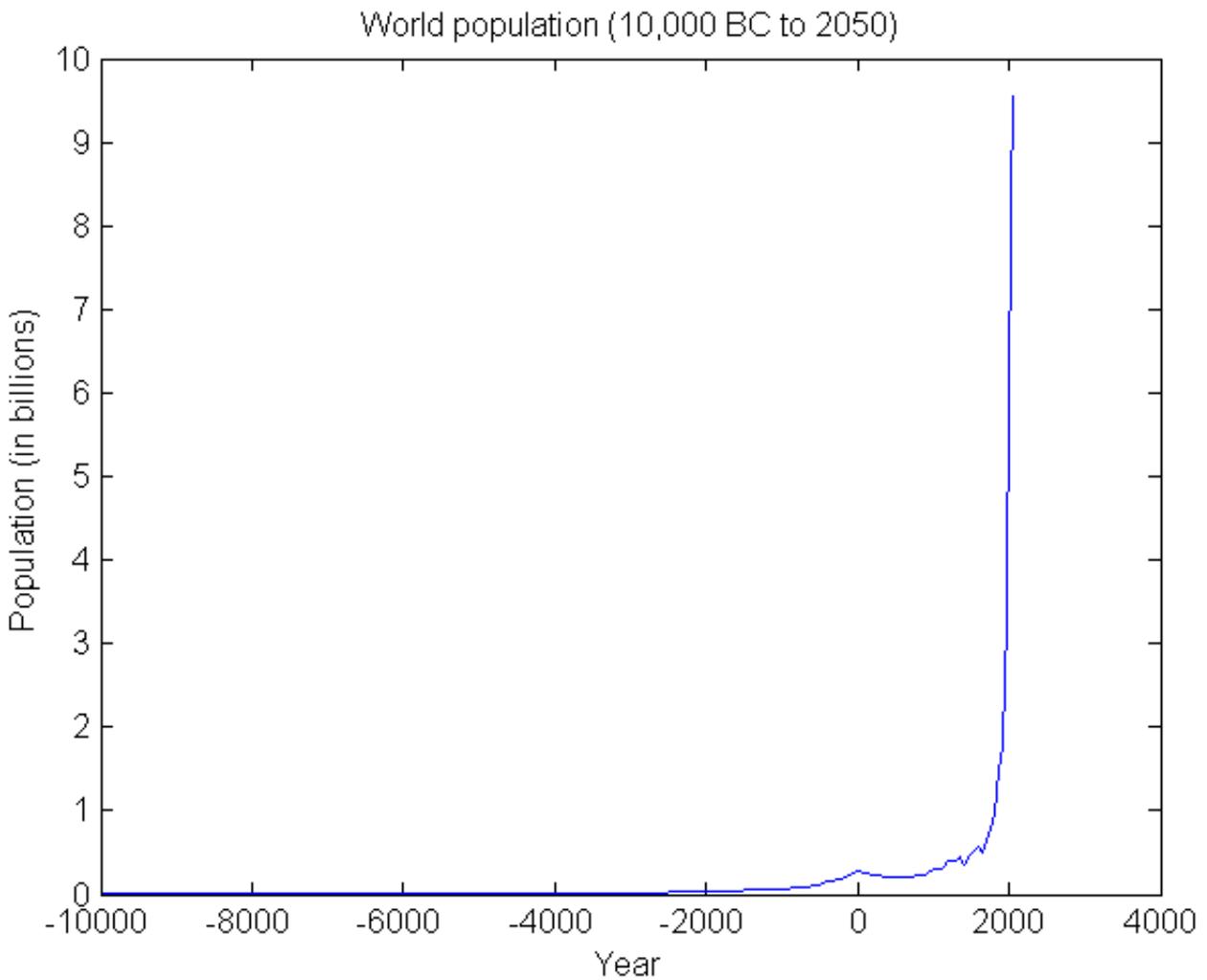
EXAMPLE 7: Read in the world population data and define variables

```
data = csvread('WorldPopulation.csv');
year = data(:, 1);
population = data(:, 2);
```

EXAMPLE 8: Plot the world population as a function of time

Create a new cell in which you type and execute:

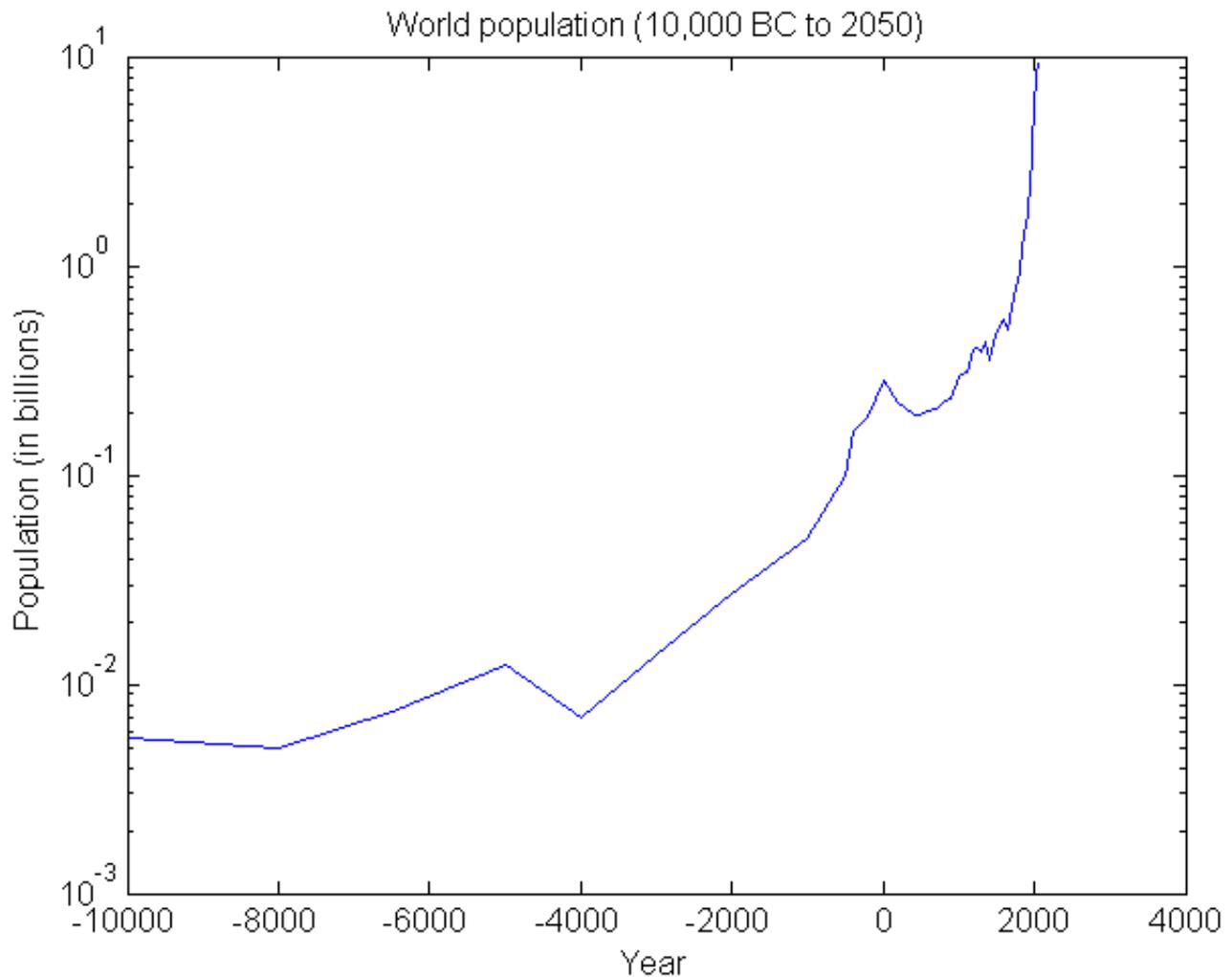
```
figure
plot(year, population)
xlabel('Year');
ylabel('Population (in billions)')
title('World population (10,000 BC to 2050)');
```



Questions	Answers
How can I interpret this graph?	It is really hard to tell much except that in recent times the population grew very fast. The problem is that both the x and y data vary over too large a range to look at on a single graph. A logarithmic transformation should help here.

EXAMPLE 9: Plot year vs. world population on a semilog scale

```
figure
semilogy(year, population)
xlabel('Year');
ylabel('Population (in billions)')
title('World population (10,000 BC to 2050)');
```



Questions	Answers
How can I interpret this graph?	This graph plots the year versus the logarithm of the population. If the population were growing exponentially, the graph would look linear. Unfortunately, this resulting graph appears to be growing "exponentially", meaning that the population growth rate is faster than exponential. (Think googel-plex rather than googel.) You will have to apply another log to find out just how bad things are.

EXAMPLE 10: Focus on the population data for 1500-2050

```

yr = year(27:end); % Pick out years 1500-2050
pop = population(27:end).*1e+9; % Convert to units of people so log of log works
popGRate = diff(pop)./diff(yr); % Annual growth rate
mYr = (yr(1:end-1) + yr(2:end))./ 2; % The yr midpoints for plotting growth rate
perCapitaGR = popGRate./pop(1:(end-1)); % Growth rate per person (birth rate)

```

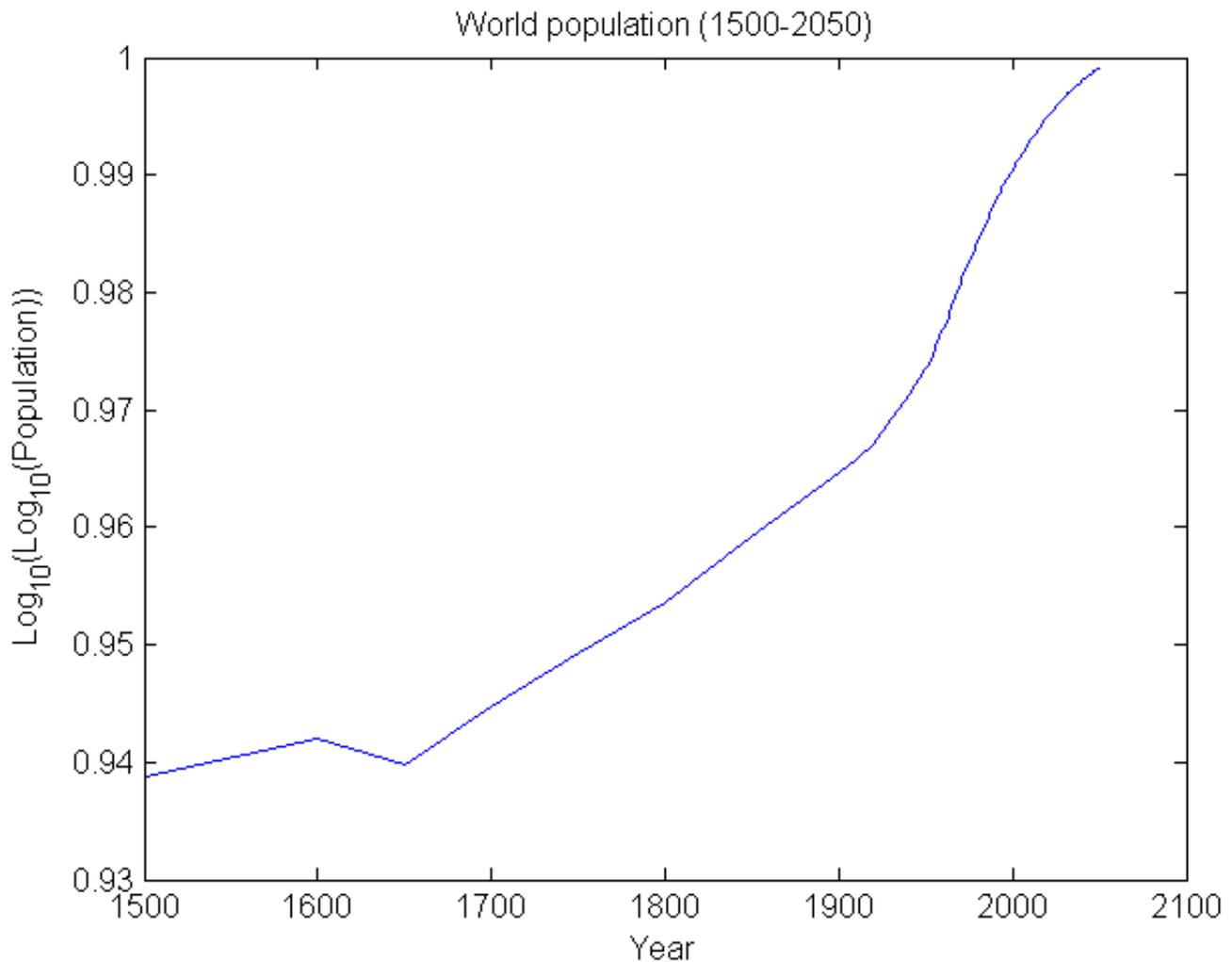
Questions	Answers
What does the per capita growth rate of a polynomial look like?	The per capita growth rate of a polynomial (power of x) decays to zero.
What does the per capita growth rate of an exponential	The per capita growth rate of an exponential should be constant (flat).

look like?	
How can I interpret the annual per capita growth rate of this graph (the green line)?	From about 1650 to 1900 the per capita growth rate was flat and had a value of about 0.005, indicating the population was growing exponentially. After 1900 the per capita growth rate started to climb (population explosion). There was a temporary plateau between 1925 and 1945 at a growth rate of 0.01, before it took off again. The per capita growth rate is now falling but it is still substantially above rates from earlier times.

EXAMPLE 11: Plot the year vs the log of the log of the world population

```
figure
plot(yr, log10(log10(pop)))
xlabel('Year');
ylabel('Log_{10}(Log_{10}(Population))') % Think about label here
title('World population (1500-2050)')
```

Questions	Answers
Why did this plot the log of the log of population?	The previous graphs showed evidence that the population grew faster than exponential. If the rate of the exponential is growing exponentially, the graph of the $\log_{10}(\log_{10}(\text{pop}))$ will appear linear (i.e., we have a googel-plex rather than merely a googel). This indeed appears to be the case.
How is $\text{plot}(x, \log_{10}(\log_{10}(y)))$ different from $\text{loglog}(x, y)$?	The $\text{plot}(x, \log_{10}(\log_{10}(y)))$ draws a graph of x versus the $\log_{10}(\log_{10}(y))$ using an ordinary scale. The $\text{loglog}(x, y)$ draws a graph of $\log_{10}(x)$ versus $\log_{10}(y)$ with both axes using logarithmic scales.



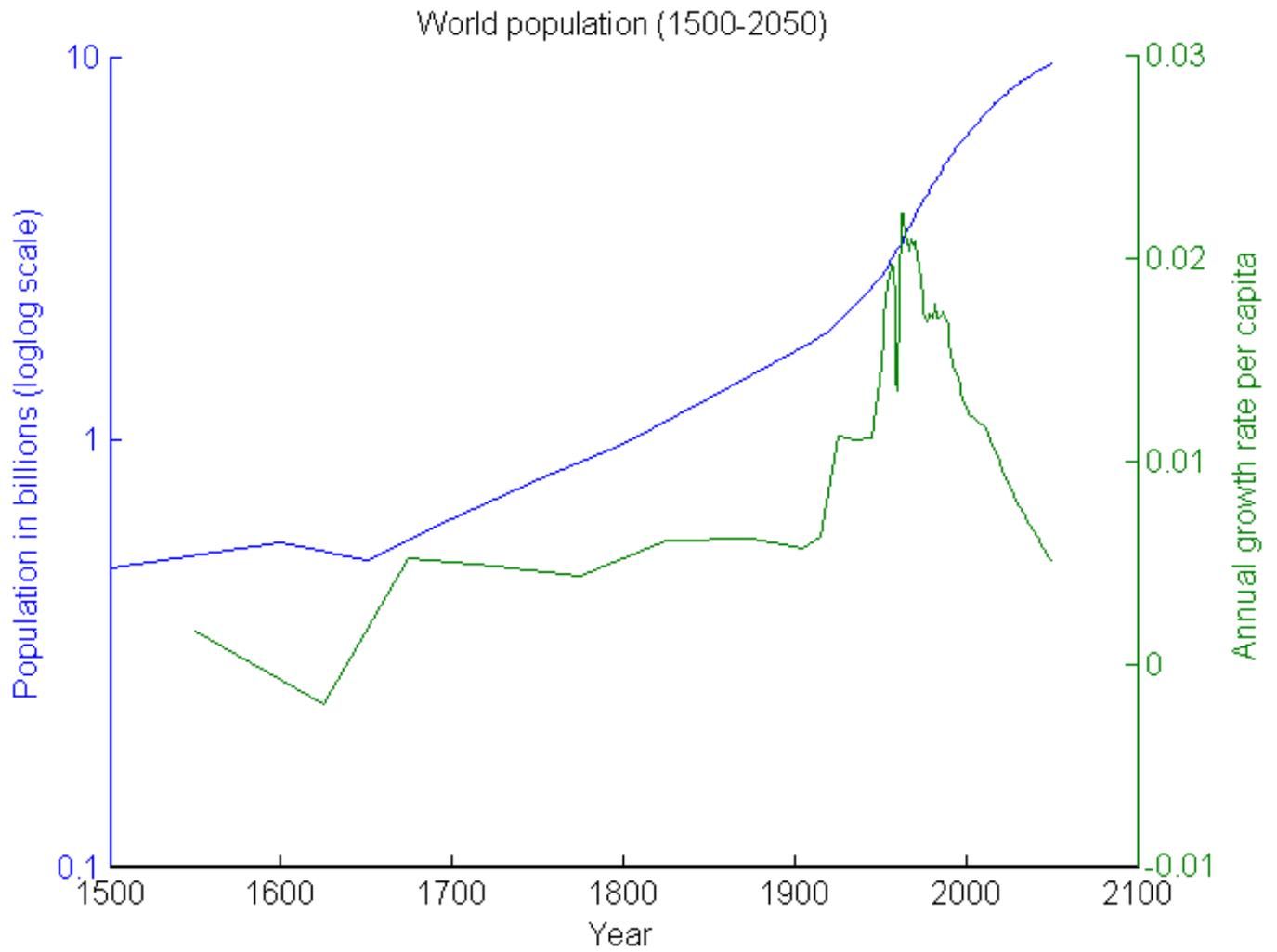
EXAMPLE 12: Plot log of the log of population versus its growth rate

Create a new cell in which you type and execute:

```
figure
ax = plotyy(yr, log10(log10(pop)), mYr, perCapitaGR);
title('World population (1500-2050)')
box off

% Set the left axis labels and tick marks manually using the ax(1) handle
xlabel(ax(1), 'Year');
ylabel(ax(1), 'Population in billions (loglog scale)')
set(ax(1), 'YLimMode', 'manual', 'YLim', [log10(8), log10(10)]);
set(ax(1), 'YTickMode', 'manual', 'YTickLabelMode', 'manual');
set(ax(1), 'YTick', [log10(8), log10(9), log10(10)]);
set(ax(1), 'YTickLabel', [0.1, 1, 10]);

% Set the right axis labels and tick marks manually using the ax(2) handle
xlabel(ax(2), ''); % Turn off x-axis label
ylabel(ax(2), 'Annual growth rate per capita') % Label right y-axis
set(ax(2), 'XTickMode', 'manual', 'XTick', []); % Turn off x ticks
```



This lesson was written by Kay A. Robbins of the University of Texas at San Antonio and last modified on 22-Apr-2016. Please contact krobbins@cs.utsa.edu with comments or suggestions.
