# Logarithms and growth rates

**Definition of the logarithm:** The value of $\log_b(x)$ is defined by the equation:

$$x = b^{\log_b x}$$

where $b$ is the base of the logarithm. In other words, $\log_b(x)$ is the power you have to raise $b$ to in order to get $x$. Base 10 or the common log base is widely used in scientific calculations, base 2 or the binary base is used in computer science, and base $e$ designates the natural log used in mathematics. Euler's number, $e$, is approximately 2.718. If you need the value of $e$ in MATLAB, use `exp(1)`.

**Logarithms and exponentials in MATLAB:** MATLAB evaluates all functions element-wise. The following MATLAB functions allow you to compute logarithms and exponentials:

- logarithm in binary base (2): `log2(X)`
- logarithm in common base (10): `log10(X)`
- logarithm in natural base ($e$): `log(X)`
- exponential in natural base ($e$): `exp(X)`
- exponential in base ($b$): `power(b, X)`

**Logarithmic plots MATLAB:** Suppose you have data in vectors `x` and `y` corresponding to the variables $x$ and $y$, respectively.

- To plot $x$ versus $\log_{10}(y)$:
  - `semilogy(x, y)` plots $y$ on a $\log_{10}$ scale, so you can label y-axis with ordinary units (e.g., grams).
  - `plot(x, log10(y))` plots $\log_{10}(y)$ on an ordinary scale, so you need to label $y$-axis with log units (e.g., $\log_{10}$(grams)).

    Alternatively, you can pick values $t_i$ of $y$ on an ordinary scale and manually set the tick mark positions at `log10(ti)`. Use tick mark labels $t_i$ and label the y-axis in ordinary units (e.g., grams). The tick marks won't be evenly spaced.

- To plot $\log_{10}(x)$ versus $\log_{10}(y)$:
  - `loglog(x, y)` plots both $x$ and $y$ on a $\log_{10}$ scale, so you can label both axes with ordinary units (e.g., weeks and grams).
  - `plot(log10(x), log10(y))` plots $\log_{10}(x)$ and $\log_{10}(y)$ on ordinary scales, so you need to label both axes with log units (e.g., $\log_{10}$(weeks) and $\log_{10}$(grams)). Alternatively you can set the tick marks for both axes as described for the y-axis in the discussion of `plot(x, log10(y))` above.

- To plot $x$ versus $\log_{10}(\log_{10}(y))$:
    - `semilogy(x, log10(y))` plots $\log_{10}(y)$ on a $\log_{10}$ scale, so you can label y-axis with logarithmic units (e.g., $\log_{10}$(grams)).
    - `plot(x, log10(log10(y)))` plots $\log_{10}(\log_{10}(y))$ on an ordinary scale, so you need to label the $y$-axis with log-log units (e.g., $\log_{10}(\log_{10}$(grams))).

      Alternatively, you can pick values $t_i$ of $y$ on an ordinary scale and manually set the tick mark positions at `log10(log10((ti)).` Use tick mark labels $t_i$ and label the y-axis in ordinary units (e.g., grams). The tick marks won't be evenly spaced.

**Testing growth rates using logarithmic scales:** Suppose you have data in vectors `x` and `y` corresponding to variables $x$ and $y$, respectively.

**Power law:** $y = x^n$
- The plot of $\log_{10}(x)$ versus $\log_{10}(y)$ appears linear and the slope is $n$.
- The plot of $x$ versus $\log_{10}(y)$ appears concave downward.

**Exponential:** $y = e^{rx}$ (or any other base)
- The plot of $\log_{10}(x)$ versus $\log_{10}(y)$ appears exponential.
- The plot of $x$ versus $\log_{10}(y)$ appears linear with slope $r$. For $y = b^{rx}$ the slope is *rlog(b).*
- The growth rate per capita: `diff(y)./diff(x)./x(1:end-1)` should be constant with value $r$.

**Faster than exponential:** $y = e^{e^{rx}}$
- The plot of $x$ versus $\log_{10}(y)$ appears exponential.
- The plot of $x$ versus $\log_{10}(\log_{10}(y))$ appears linear (for this rate of growth at least).

| **Logarithmic identity** | **Corresponding exponential identity** |
|---|---|
| $\log_b(1) = 0$ | $b^0 = 1$ |
| $\log_b(b) = 1$ | $b^1 = b$ |
| $\log_b(b^x) = x$ | $b^{\log_b x} = x$ |
| $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$ | $b^x \cdot b^y = b^{x+y}$ |
| $\log_b(x/y) = \log_b(x) - \log_b(y)$ | $b^x / b^y = b^{x-y}$ |
| $\log_b(x^y) = y \log_b(x)$ | $(b^x)^y = b^{xy}$ |
| $\log_b(\sqrt[y]{x}) = \log_b(x)/y$ | $\sqrt[y]{x} = x^{1/y}$ |
| $x^{\log_b y} = y^{\log_b x}$ | $x^{\log_b y} = b^{\log_b x \log_b y}$ |
| $\log_a(b) = \log_c(b) / \log_c(a)$ (change of base) | |

$b$, $x$, $y$ are positive and $b \neq 1$