

# 3343 (Spring 2008) Assignment 1

## Solution

1. (20 points) Order the following functions according to their order of growth from the lowest to the highest. If you think that two functions are of the same order (i.e.  $f(n) = \Theta(g(n))$ ). Then put them in the same group.

$\log(n + 100), n^n, 10^n, n^2, (n + 1)!, n, 0.01n^4 + 3n^3 + 1, \log n + 100n, n^2 \log n.$

**The right answer is:**

$\log(n + 100)$

$n \sim \log n + 100n$

$n^2$

$n^2 \log n$

$0.01n^4 + 3n^3 + 1$

$10^n$

$(n + 1)!$

$n^n$

2. (20 points) Show that, if  $c$  is a positive real number, then  $g(n) = 1 + c + c^2 + \dots + c^n$  is:

- $\Theta(1)$  if  $c < 1$ .
- $\Theta(n)$  if  $c = 1$ .
- $\Theta(c^n)$  if  $c > 1$ .

**Proof:**

Note that the terms in  $g(n)$  form a geometric series. Therefore, the sum can be computed by

$$g(n) = \sum_{i=0}^n c^i = \begin{cases} \frac{c^{n+1}-1}{c-1} & \text{if } c \neq 1; \\ n+1 & \text{if } c = 1. \end{cases}$$

- If  $c < 1$ ,  $\lim_{n \rightarrow \infty} c^{n+1} = 0$ , therefore  $\lim_{n \rightarrow \infty} \frac{g(n)}{1} = \lim_{n \rightarrow \infty} g(n) = \frac{1}{1-c}$ , which is a positive constant. Therefore,  $g(n) \in \Theta(1)$ .
- If  $c = 1$ ,  $g(n) = n + 1 \in \Theta(n)$ .
- If  $c > 1$ ,  $\lim_{n \rightarrow \infty} \frac{g(n)}{c^n} = \lim_{n \rightarrow \infty} \frac{c^{n+1}-1}{(c-1)c^n} = \frac{1}{c-1} \lim_{n \rightarrow \infty} (c - \frac{1}{c^n}) = \frac{c}{c-1}$ , which is a positive constant. Therefore,  $g(n) \in \Theta(c^n)$ .

3. (20 points) Using the basic definition method, show that  $10n^2 = O(n^3)$ .

**Proof:**

Let  $c = 1$ , and  $n_0 = 10$ . For all  $n > n_0$ , we have  $10 \leq cn$ . Hence  $10n^2 \leq c n n^2 = cn^3$  for all  $n > n_0$ . In addition,  $10n^2 \geq 0$  for all  $n > n_0$ . Therefore, by definition,  $10n^2$  is in  $O(n^3)$ .