

# CS 3343 (Spring 2008) Assignment 2

## Solution

1. (10 points) Order the following functions according to their order of growth from the lowest to the highest. If you think that two functions are of the same order (i.e.  $f(n) \in \Theta(g(n))$ ), put them in the same group.

$$\log^2 n, (\log \log n)^2, \log \log(n!), n/\log n, \sqrt{n}, \log n$$

**Solution:** It is easy to obtain  $\log n \ll \log^2 n \ll \sqrt{n} \ll n/\log n$  from the lecture. We just need to figure out where should we place  $(\log \log n)^2$  and  $\log \log(n!)$ . First, we know that  $\log(n!) \in \Theta(n \log n)$ . Therefore  $\log \log(n!) \in \Theta(\log(n \log n)) \in \Theta(\log n + \log \log n) \in \Theta(\log n)$ . Second, we know that  $(\log \log n)^2 \in o(\log^2 n)$ , since  $\log \log n \in o(\log n)$ . So we need to compare  $(\log \log n)^2$  with  $\log n$ . We can change variable to let  $m = \log n$ . We then have  $(\log \log n)^2 = \log^2 m \in o(m)$ . Substitute  $m$  with  $\log n$  we have  $(\log \log n)^2 \in o(\log n)$ .

**Therefore, the right order should be:**

$$(\log \log n)^2 \ll \log n \sim \log \log(n!) \ll \log^2 n \ll \sqrt{n} \ll n/\log n$$

2. (30 points) For each pair of functions in the table below, determine whether  $f(n) \in O(g(n))$ ,  $f(n) \in \Omega(g(n))$ , or both (i.e.  $f(n) \in \Theta(g(n))$ ). Complete the table following the examples in the first three lines.

	$f(n)$	$O$	$\Omega$	$\Theta$	$g(n)$
i.	$n^2$	x			$n^3$
ii.	$n$		x		$\log n$
iii.	$n^2 - n$	x	x	x	$10n^2 + 6n$
a.	$n^2 - n$		X		$6n$
b.	$n + 2\sqrt{n}$	X			$n^2$
c.	$n + n \log n$	X			$n\sqrt{n}$
d.	$n + \log n$		X		$\sqrt{n}$
e.	$n^2 + 3n + 4$	X			$n^3$
f.	$2^n$		X		$3^{n/2}$
g.	$\log(n^2)$	X			$n \log n$
h.	$2 \log^2 n$		X		$\log n + 10$
i.	$\log(n!)$	X	X	X	$\log(n^{n/2})$
j.	$\log_{10}(n + 10)$	X	X	X	$\log_2(2n + 5)$

3. (20 points) Using **the basic definition method**, prove that  $n + 10\sqrt{n} \in O(n)$ .

**Proof scratch:** We have to solve  $0 \leq n + 10\sqrt{n} \leq cn$ . The first inequality is easy to satisfy as long as  $n \geq 0$ . Solving the second part, we obtain  $(c - 1)\sqrt{n} \geq 10$ . You get the credit if you have chosen any  $c$  and  $n_0$  such that the above inequality is true for all  $n > n_0$ . For example  $c = 2$ , and  $n_0 = 100$  or  $c = 11$  and  $n_0 = 1$ . You can not choose any  $c$  that is smaller than or equal to 1, for then  $n < n + 10\sqrt{n}$  no matter how big  $n$  is.

**Proof:** Let  $c = 11$  and  $n_0 = 1$ . First,  $n + 10\sqrt{n} \geq 0$  for all  $n > 1$ . Second,  $11n \geq n + 10\sqrt{n}$  for all  $n > 1$ , because  $10n > 10\sqrt{n}$ . Therefore, according to definition,  $n + 10\sqrt{n} \in O(n)$ .