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PAPER

Heuristic and Exact Algorithms for QoS Routing with Multiple Constraints*

Gang FENG[†], Kia MAKKI[†], Niki PISSINOU[†], *and* Christos DOULIGERIS[††], *Nonmembers*

SUMMARY The modern network service of finding the optimal path subject to multiple constraints on performance metrics such as delay, jitter, loss probability, etc. gives rise to the multi-constrained optimal-path (MCOP) QoS routing problem, which is $\mathcal{NP}$-complete. In this paper, this problem is solved through both exact and heuristic algorithms. We propose an exact algorithm E_MCOP, which first constructs an aggregate weight and then uses a K-shortest-path algorithm to find the optimal solution. By means of E_MCOP, the performance of the heuristic algorithm H_MCOP proposed by Korkmaz et al. in a recent work is evaluated. H_MCOP only runs Dijkstra's algorithm (with slight modifications) twice, but it can find feasible paths with a success ratio very close to that of the exact algorithm. However, we notice that in certain cases its feasible solution has an unsatisfactorily high average cost deviation from the corresponding optimal solution. For this reason, we propose some modified algorithms based on H_MCOP that can significantly improve the performance by running Dijkstra's algorithm a few more times. The performance of the exact algorithm and heuristics is investigated through computer simulations on networks of various sizes.
key words: *QoS routing, multi-constrained path, additive QoS constraints, K-shortest-path algorithm, high-speed networking*

1. Introduction

Due to the increasing need for providing modern network services such as VoIP, video and interactive multimedia communications, quality-of-service (QoS) routing has attracted much attention in the past few years [20]. The major task of QoS routing is to identify a path or a tree that satisfies one or more QoS requirements. In case of unicast routing we need to find a path between a source and a single destination [3], while in case of multicast routing, we need to find a tree that spans a source and a group of destinations [17]. In terms of the responsibility of each node involved in the computing of the path (or the tree), QoS routing can also be classified into source routing [3] and distributed routing [21], [22]. In this paper, we restrict our interest to QoS unicast source routing, which means that the source node is going to take full responsibility for finding a qualified route to the given destination. We assume that a link-state routing protocol such as *open shortest path first* (OSPF) [18] is available to provide all necessary state information to the source node.

QoS requirements are generally specified in terms of constraints imposed upon the corresponding performance metrics [23]. In [3], these constraints are roughly classified into *link constraint* and *path constraint*, and based on these concepts, the QoS unicast routing problems are divided into several categories, among which the *path-constrained path-optimization* (PCPO) problem and the *multiple-path-constrained* (MCP) problem are $\mathcal{NP}$-complete [10] and have become a hot topic in the QoS routing community in recent years.

Most previous works for the PCPO and the MCP problems only consider the cases when the number of constraints is small (≤ 2). For instance, as the most representative PCPO problem, the delay-constrained least-cost (DCLC) problem has received extensive studies, and consequently a number of exact and heuristic algorithms have been developed [7], [11], [13], [20]–[22], [25]. On the other hand, many researchers also studied the delay-cost-constrained (DCC) problem [2], [8], [12], which is an MCP instance with two constraints. For a complete overview of these algorithms published before the year of 2000, the readers are referred to [14].

In spite of the good performance of some of the prior works, in the past two or three years a few authors [7], [13], [14], [19] found that the heuristic algorithms based on the Lagrange relaxation are much more attractive either in the sense of the time complexity or from the quality of solution standpoint. For example, Jüttner et al. [13] and Feng et al. [7] almost at the same time proposed the same iterative algorithm for the DCLC problem, which is based on the linear Lagrange relaxation. The basic idea of this algorithm is to linearly combine the delay and the cost in terms of a parameter to form an aggregate weight w, and then use the Dijkstra's algorithm [1], [4] to find the shortest path w.r.t. w. By tuning the parameter according to the path obtained in the previous iteration, the algorithm tries to find a good feasible path if there exists such one. Large quantities of experiments indicate that this algorithm can obtain the optimal[†] solution with

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[†]The authors are with Telecommunications & Information Technology Institute, Florida International University, Miami, FL 33174, USA.
[††]The author is with the Department of Informatics, University of Piraeus, Piraeus 18534, Greece.
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a very high probability even when the network size is relatively large [7], and most importantly its time complexity is very low [7], [13]. Feng et al. further extended this algorithm to solve the DCC problem [8], and it also outperforms most existing algorithms.

Our recent research results [9] demonstrate that, however, when the number of performance metrics considered is greater than two, e.g., the PCPO problem with two or more constraints or the MCP problem with three or more constraints, it is hard to develop efficient heuristic algorithms based on the linear Lagrange relaxation technique. Instead, the heuristic algorithm H_MCOP proposed by Kormaz et al. [14], which is based on the nonlinear Lagrange relaxation, delivers much better performance. The difference between the linear and nonlinear Lagrange relaxation was illustrated in [14]. While the linear relaxation method might be trapped by an infeasible path, the nonlinear relaxation method can possibly search the whole feasible region. The major difficulty of the nonlinear relaxation lies in that we can not use Dijkstra's algorithm to find a path that exactly minimizes the objective function. For more details, please see [14]. To attack this difficulty, H_MCOP runs Dijkstra's algorithm (with modifications) twice, one in reverse direction with linear relaxation and the other in forward direction with nonlinear relaxation. It has been proved through computer simulations that in case of two constraints H_MCOP can outperform almost all known algorithms either in terms of the success ratio of finding feasible paths or in terms of the average cost of the obtained feasible paths.

H_MCOP was designed to solve the multi-constrained optimal-path (MCOP) problem, i.e., PCPO problem with multiple constraints. Despite of its excellent performance shown in [14] when compared with other heuristics, the "goodness" of the obtained solution is still unclear in comparison with an optimal solution. For this reason, we investigate the performance of H_MCOP in this paper by comparing it with an exact algorithm in various situations. As a result, we propose two exact algorithms for the MCP problem and the MCOP problem, respectively. Previous works on the exact algorithms include the extended Belman-Ford (EBF) algorithm [27] for the MCP problem and the A*Prune for the MCOP problem [16]. Compared with these algorithms, the exact algorithms described in this paper are easier to implement if one has a loopless K-shortest-path (KSP) algorithm ready for use. Computer simulations also demonstrate they are not very time consuming when the network size is not very large (e.g., ≤ 200 nodes).

The second contribution of this paper is some modified algorithms based on H_MCOP. Our experimental results indicate that even though the success ratio for finding feasible paths by H_MCOP is very close to that of the exact algorithm, in certain cases the average cost deviation between the obtained feasible solution and the optimal solution is not low enough to be satisfied. The modified algorithms described in this paper, however, can significantly improve its performance with slightly increased time complexity.

The remainder of this paper is organized as follows. In Sect. 2, the MCOP problem is formally defined. In Sect. 3 two exact algorithms are described for the MCP and MCOP problems, respectively. A modified algorithm and its possible variants based on H_MCOP are discussed in Sect. 4. The performance evaluation is given in Sect. 5, and Sect. 6 concludes this paper.

2. Notations and Definitions

Since the modified algorithm to be proposed is based on H_MCOP, most notations used in this paper are the same as in [14]. A network is represented by a digraph $G(V, E)$, where V is the set of nodes and E is the set of links. Associated with each link e there are J non-negative weights $w_j(e), j = 0, 1, \cdots, J-1$ and a cost $c(e)$. Each weight is corresponding to a constraint, and the upper bound of constraint j is denoted by C_j.

A path is a sequence of non-repeated nodes $\mathbf{p} = (v_1, v_2, \cdots, v_k)$ such that for a given $1 \leq i < k$ there exists a link from v_i to v_{i+1}. The notation $e \in \mathbf{p}$ means that path $\mathbf{p}$ passes through link e. The w_j-weight and cost of path $\mathbf{p}$ are given by

$$w_j(\mathbf{p}) = \sum_{e \in \mathbf{p}} w_j(e) \text{ and } c(\mathbf{p}) = \sum_{e \in \mathbf{p}} c(e),$$

respectively.

By means of the above notations we define the multi-constrained optimal-path (MCOP) problem as follows.

Definition 1: Given a routing request between a source s and a destination t, the multi-constrained optimal-path problem is to find a path $\mathbf{p}$ between s and t such that

(i) $w_j(\mathbf{p}) \leq C_j, \forall j = 0, 1, \cdots, J-1$
(ii) $c(\mathbf{p}) \leq c(\mathbf{q})$ for any path $\mathbf{q}$ that satisfies (i).

A path satisfying (i) is called a *feasible path* (or *feasible solution*), and otherwise an *infeasible path* (or *infeasible solution*). If a path satisfies both (i) and (ii), it is called an optimal solution.

Without considering part (ii) in the above definition, the corresponding problem becomes the MCP problem, for which we are only concerned with the question whether there is a feasible path or not.

In order to describe the exact algorithms for the

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†For the PCPO problem, there may exist more than one optimal solution. Therefore in this paper "the optimal solution" does not necessarily mean that there is only one optimal solution.

MCP and MCOP problems in the next Section, we specially denote the shortest path between s and t w.r.t. w_j by $\mathbf{p}_j$, and the least cost (LC) path by $\mathbf{p}_c$.

3. Exact Algorithms for the MCP and MCOP Problems

As mentioned in Sect. 1, both the MCP problem and the MCOP problem are $\mathcal{NP}$-complete. Thus, any exact algorithm for these problems generally can not be used in practical applications. However, an exact algorithm is very useful to evaluate the performance of other heuristic algorithms. In this Section, we describe two exact algorithms E_MCP and E_MCOP for the above two problems, respectively. Algorithm E_MCP can find a feasible path for the MCP problem if there exists such one, while algorithm E_MCOP can return the optimal path for the MCOP problem if there exists at least one feasible path.

3.1 Algorithm E_MCP

Our basic idea of the exact algorithm for the MCP problem is first to form an aggregate weight $w(e)$ for each link e, and then use a loopless KSP algorithm [15], [26] to check the shortest paths w.r.t. w between s and t. Since the KSP algorithm can find the shortest path, the second shortest path, …, until there is no more path available between s and t, we definitely can find a feasible path as long as there exists at least one. The worst case is to enumerate all paths between s and t. The aggregate weight $w(e)$ for link e is a linear combination of the J weights, given by

$$w(e) = w_0(e) + \sum_{j=1}^{J-1} \beta_j w_j(e) \qquad (1)$$

where $\beta_j, j = 1, 2, \cdots, J-1$ are positive numbers. The reason to check the shortest paths w.r.t. the aggregate weight w instead of an individual weight is because by doing so we can possibly significantly reduce the number of paths to be checked before a feasible path is found or a conclusion of unavailability of a feasible path is reached.

The above idea can be illustrated by a simple example. For the MCP problem described in Fig. 1(a), we assume that there are three paths between s and t, and there are three weights associated with each link (or path). The upper bounds of the three constraints are assumed to be 5. Obviously path $\mathbf{q}_1$ is the only feasible path. If we use a KSP algorithm to check the shortest paths w.r.t. any one of the three weights, path $\mathbf{q}_1$ must be the second shortest path. However, if we construct the aggregate weight w with $\beta_1 = \beta_2 = 1$, the w-weights of the three paths are shown in Fig. 1(b), from which we can see path $\mathbf{q}_1$ is the shortest path.

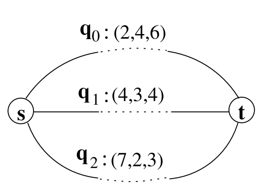
There are two critical issues in the exact algorithm based on the above idea. First, what parameter should be chosen to construct the aggregate weight so that a feasible path can be found as early as possible? Second, when can we conclude that no feasible path is available? For a large network, there may have millions of paths between any two nodes. In that case, if there is no feasible path at all, definitely it is very time consuming and unwise to check all paths.

Regarding the choice of parameters $\beta_j, j = 1, 2, \cdots, J-1$, we recommend using the following method. Assuming that $C_j \geq w_j(\mathbf{p}_j), \forall j = 0, 1, \cdots, J-1$, we first try to find a weight w_j such that $C_j > w_j(\mathbf{p}_j)$. If there does not exist such a weight (meaning $C_j = w_j(\mathbf{p}_j), \forall j = 0, 1, \cdots, J-1$), let $\beta_j = 1, \forall j = 1, 2, \cdots, J-1$. Otherwise, without loss of generality we assume $C_0 > w_0(\mathbf{p}_0)$, and let β_j be given by

$$\beta_j = \begin{cases} \infty & : \quad C_j = w_j(\mathbf{p}_j) \\ \dfrac{C_0 - w_0(\mathbf{p}_0)}{C_j - w_j(\mathbf{p}_j)} & : \quad \text{otherwise} \end{cases}$$
$$j = 1, 2, \cdots, J-1.$$

For implementation, ∞ in the above formula can be replaced by a large number.

The above method of choosing β_j can ensure that the change of the aggregate weight has the equal effect on any constraint. This point is illustrated through Fig. 2, in which we assume there are two weights associated with each link, and hence we need to choose a value for β_1. Each black dot denotes a path with its w_0-weight and w_1-weight being the horizontal and vertical coordinates, respectively. The shaded areas represent the feasibility regions. Any two paths located on the same contour line have the same value w.r.t. the ag-



Aggregate weight for each path (β1=β2=1)		
$\mathbf{q}_0$: 12	$\mathbf{q}_1$: 11	$\mathbf{q}_2$: 12

(a) (b)

Fig. 1 A MCP problem with upper bounds of 5.

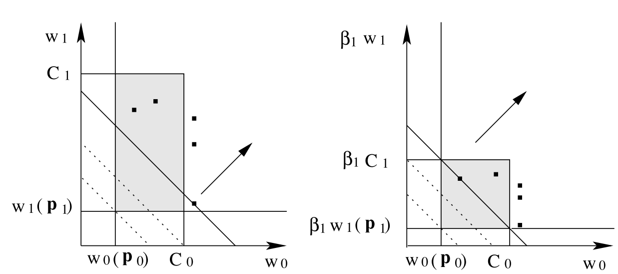


Fig. 2 Search for a feasible path.

gregate weight w. The KSP algorithm will return the path in the order being hit by the contour line sliding from the origin along the perpendicular direction. Figure 2(a) shows the case when $\beta_1 = 1$, while Fig. 2(b) shows the case when β_1 is computed by means of the method described above (assume $\beta_1 \neq 1$ in the latter case). Since in the latter case the feasibility region is a square, the KSP can search at least half of the feasibility region before it returns an infeasible path, and thus significantly reduce the number of paths to be checked before a conclusion is made. One can imagine that in the case of more constraints, the feasibility region becomes a hypercube with equal length for each edge if β_j is computed using the above method.

Regarding the second question, we have the following proposition.

Proposition 1: Given a path $\mathbf{r}$, if $w_0(\mathbf{r}) + \sum_{j=1}^{J-1} \beta_j w_j(\mathbf{r}) > C_0 + \sum_{j=1}^{J-1} \beta_j C_j$, then $\mathbf{r}$ must be an infeasible path.

Proof: If we assume that path $\mathbf{r}$ satisfies $J-1$ constraints, then the remaining one must be violated. □

This proposition implies that $MCP_{max} = C_0 + \sum_{j=1}^{J-1} \beta_j C_j$ is the upper limit on the aggregate weight for a path to be feasible. When using the KSP algorithm to check the shortest paths one by one, if the w-weight of the i-th shortest path exceeds MCP_{max}, we can conclude that all of the subsequent shortest paths must be infeasible.

Based on the above analysis, we can develop an exact algorithm E_MCP for the MCP problem. As shown in Fig. 3, this algorithm starts by finding the shortest path $\mathbf{p}_j$ w.r.t. each weight $w_j, \forall j = 0, 1, \cdots, J-1$. If $C_j < w_j(\mathbf{p}_j)$ for any j, then the algorithm stops since it is impossible to find a feasible path. Otherwise, β_j, w and MCP_{max} are computed based on our above description, and a loopless KSP algorithm is used to check the shortest paths w.r.t. w between s and t one by one. If the current shortest path $\mathbf{r}$ is a feasible path, then the algorithm terminates with $\mathbf{r}$ returned. If there is no more path between s and t, or if the w-weight of the current shortest path $\mathbf{r}$ exceeds MCP_{max}, we can also conclude that there is no feasible path. Otherwise, check the next shortest path. If the algorithm can not find a feasible path, a NULL pointer is returned.

E_MCP$(G(V, E), s, t, w_j, C_j, j = 0, 1, \cdots, J-1)$
Find path $\mathbf{p}_j, \forall j = 0, 1, \cdots, J-1$.
If $(C_j < w_j(\mathbf{p}_j))$ for any j **then**
    return NULL /* no feasible path available */
Compute $\beta_j, \forall j = 1, 2, \cdots, J-1$
Compute $w(e)$ for each link e according to equation (1)
set $MCP_{max} = C_0 + \sum_{j=1}^{J-1} \beta_j C_j$
repeat
    $\mathbf{r} \leftarrow$the current shortest path w.r.t. w found by a KSP algorithm
    if $(\mathbf{r} = \text{NULL})$ **then**
        return NULL /* no more path */
    if $(w_0(\mathbf{r}) + \sum_{j=1}^{J-1} \beta_j w_j(\mathbf{r}) > MCP_{max})$ **then**
        return NULL
    if $(w_j(\mathbf{r}) \leq C_j, \forall j = 0, 1, \cdots, J-1)$ **then**
        return $\mathbf{r}$

Fig. 3 The exact algorithm E_MCP for the MCP problem.

3.2 Algorithm E_MCOP

The exact algorithm for the MCOP problem is based on the same idea as that for the MCP problem, i.e., we first construct an aggregate weight $w(e)$ for each link e, and then use a loopless KSP algorithm to find the optimal solution. However, some details have to be changed since in this case we need to find a path with the minimal cost among all feasible paths.

In order to take into account the cost c, the aggregate is given by

$$w(e) = \alpha c(e) + w_0 + \sum_{j=1}^{J-1} \beta_j w_j(e) \tag{2}$$

where α and $\beta_j, j = 1, 2, \cdots, J-1$ are positive numbers. The parameters $\beta_j, j = 1, 2, \cdots, J-1$ can be chosen in the same way as in the MCP case. Parameter α can be chosen as follows. If $C_j = w_j(\mathbf{p}_j), \forall j = 1, 2, \cdots, J-1$, we let $\alpha = 1$. Otherwise, we assume that $C_0 > w_0(\mathbf{p}_0)$ and $\mathbf{h}$ is a feasible path with $c(\mathbf{h}) > c(\mathbf{p}_c)$, and hence we let α be given by

$$\alpha = \frac{C_0 - w_0(\mathbf{p}_0)}{c(\mathbf{h}) - c(\mathbf{p}_c)}.$$

The way for choosing the value of α is based on the same philosophy as for choosing β_j, i.e., to adjust all weights to the same level as w_0 so that the change of the aggregate weight has almost the same impact on any constraint.

Similar to the MCP case, for E_MCOP we need to find a rule according to which we can judge whether there is a better solution or not. Regarding this issue, we have a similar proposition.

Proposition 2: Assume that $\mathbf{h}$ is the best feasible solution obtained so far, and $\mathbf{r}$ is the current shortest path. If $\alpha c(\mathbf{r}) + w_0(\mathbf{r}) + \sum_{j=1}^{J-1} \beta_j w_j(\mathbf{r}) > \alpha c(\mathbf{h}) + C_0 + \sum_{j=1}^{J-1} \beta_j C_j$, then $\mathbf{h}$ must be the optimal solution.

Proof: Assuming that $\mathbf{r}$ is a feasible solution, we must have $c(\mathbf{r}) > c(\mathbf{h})$, which means that path $\mathbf{r}$ is worse than $\mathbf{h}$. □

With the above preparation, the exact algorithm E_MCOP for the MCOP problem is described in Fig. 4. As we see, this algorithm first tries to find a feasible solution by means of algorithm E_MCP without considering the cost c. If no feasible solution exists, the algorithm terminates, otherwise denote the feasible path by $\mathbf{h}$. If $c(\mathbf{h}) = c(\mathbf{p}_c)$, then the algorithm also terminates with $\mathbf{h}$ returned as the optimal solution. Otherwise,

E_MCOP$(G(V,E), s, t, c, w_j, C_j, j = 0, 1, \cdots, J-1)$
Step 1: Find a feasible path.
$\mathbf{h} \leftarrow$ E_MCP$(G(V,E), s, t, w_j, C_j, j = 0, 1, \cdots, J-1)$
if ($\mathbf{h}$ = NULL) **then**
    return NULL
if $(c(\mathbf{h}) = c(\mathbf{p}_c))$ **then**
    return $\mathbf{h}$
Step 2: Find the optimal path.
Compute α
Compute $w(e)$ for each link e according to equation (2)
set $MCOP_{max} = \alpha c(\mathbf{h}) + C_0 + \sum_{j=1}^{J-1} \beta_j C_j$
repeat
    $\mathbf{r} \leftarrow$the current shortest path w.r.t. w found by a KSP algorithm
    if ($\mathbf{r}$ = NULL) **then**
        return $\mathbf{h}$ /* no more path */
    if $(w(\mathbf{r}) > MCOP_{max})$ **then**
        return $\mathbf{h}$ /* $w(\mathbf{r})$ is given by equation (2) */
    if $(c(\mathbf{r}) < c(\mathbf{h})$ and $w_j(\mathbf{r}) \le C_j, \forall j)$ **then**
        $\mathbf{h} \leftarrow \mathbf{r}$
        update $MCOP_{max}$

Fig. 4 The exact algorithm E_MCOP for the MCOP problem.

α, $MCOP_{max}$ and w are computed in accordance with the above description (note that $\beta_j, \forall j = 1, 2, \cdots, J-1$ has the same values in E_MCP and E_MCOP). Subsequently, a loopless KSP algorithm is employed to check the shortest paths w.r.t. w one by one. If there is no more path between s and t, or if the w-weight of the current shortest path $\mathbf{r}$ is greater than $MCOP_{max}$, then the algorithm terminates with path $\mathbf{h}$ returned as the optimal solution. If the current shortest path $\mathbf{r}$ is a feasible solution with a lower cost than $\mathbf{h}$, then replace $\mathbf{h}$ by $\mathbf{r}$, update $MCOP_{max}$, and continue to check the next shortest path.

4. A Modified Heuristic for the MCOP Problem

Most probably H_MCOP [14] is so far the best heuristic algorithm for the MCOP problem, not only because of its low time complexity, but also due to its high success ratio of finding feasible solutions. In terms of the quality (cost) of the solution, it also considerably outperforms its predecessor TAMCRA [19]. In spite of this, in some cases its solution on average has a much higher cost than the corresponding optimal solution. In this section we describe a modified algorithm based on H_MCOP that can significantly improve its performance with slightly increased time complexity. Before doing this, we describe a heuristic for the MCP problem which is also based on H_MCOP.

4.1 Heuristic Algorithm H_MCP

As described in Appendix, H_MCOP runs Dijkstra's algorithm (with slight modifications on the relaxation procedures) twice: One in reverse direction with a linear cost function, and the second in forward direction with a nonlinear cost function. Even though H_MCOP can be directly used to solve an MCP problem by skipping all codes regarding the cost c, it does not achieve the best performance due to the particularity of the MCP problem. Given an MCP problem, we are only concerned with whether there exists a feasible solution or not. This differs from the case of solving an MCOP problem for which we need to find the best solution. Thus, if an algorithm for the MCP problem finds a feasible path, it may stop immediately with the path returned.

H_MCP$(G(V,E), s, t, w_j, C_j, j = 0, \cdots, J-1)$
1 Reverse_Dijkstra $(G(V,E), t, w_j, j = 0, \cdots, J-1)$
2 **if** $(r[s] > J)$ **then**
3   return NULL /* no feasible path available */
4 **if** $(R_j[s] \le C_j, \forall j = 0, \cdots, J-1)$ **then**
5   return the path found by Reverse_Dijkstra
6 Look_Ahead_Dijkstra$(G(V,E), s, w_j, C_j, j = 0, \cdots, J-1)$
7 **if** $(G_j[t] \le C_j, \forall j = 0, \cdots, J-1)$ **then**
8   return the path found by Look_Ahead_Dijkstra
9 return NULL

Fig. 5 The heuristic algorithm H_MCP for the MCP problem.

For this reason, we may make slight modifications on H_MCOP to obtain a heuristic algorithm H_MCP for the MCP problem, which is shown in Fig. 5. Comparing Fig. 5 and Fig. A·1 (in Appendix), one can see that H_MCP is very similar to H_MCOP. The major difference lies in that if a feasible path is found after calling Reverse_Dijkstra, H_MCP will stop and return this path (lines 4-5 in Fig. 5).

Subroutine Reverse_Dijkstra in H_MCP, together with its relaxation procedure, is the same as that in H_MCOP. Subroutine Look_Ahead_Dijkstra in H_MCP is slightly different from the one in H_MCOP in that the relaxation procedure for the former, which is shown in Fig. 6, does not contain those codes for processing the cost c (lines 2 and 7 in Fig. A·3). For the same reason, procedure Prefer_the_best for H_MCP shown in Fig. 7 is also slightly different from the procedure for the H_MCOP shown in Fig. A·4.

Apparently, H_MCP can achieve the same success ratio of finding feasible solutions as H_MCOP does. However, unlike H_MCOP which needs to run Dijkstra's algorithm twice, H_MCP could run only once if a feasible path is found by subroutine Reverse_Dijkstra.

4.2 Heuristic Algorithm Modified_H_MCOP

The pseudocode of heuristic Modified_H_MCOP is shown in Fig. 8. As we see, this algorithm is based on H_MCOP and H_MCP. Assuming that H_MCOP returns a NULL pointer if no feasible path is found, Modified_H_MCOP first calls H_MCOP to obtain an initial path $\mathbf{r}$. If $\mathbf{r}$ is not NULL, then the MCOP problem is converted to an MCP problem with $J+1$ constraints by setting cost c to weight J and $c(\mathbf{r}) - \epsilon$ to the upper bound C_J of constraint J. ϵ is a small pos-

Look_Ahead_Dijkstra_Relax(u, v)
Let tmp be a temporary node
if $\lambda < \infty$ **then**
$g[tmp] = \sum_{j=0}^{J-1} \left(\frac{G_j[u]+w_j(u,v)+R_j[v]}{C_j} \right)^{\lambda}$
if $\lambda = \infty$ **then**
$g[tmp] = \max \left\{ \frac{G_j[u]+w_j(u,v)+R_j[v]}{C_j} | 0 \leq j \leq J-1 \right\}$
$G_j[tmp] = G_j[u] + w_j(u,v)$ for $j = 0, 1, \ldots, J-1$
$R_j[tmp] = R_j[v]$ for $j = 0, 1, \ldots, J-1$
if (Prefer_the_best$(tmp, v) = tmp$) **then**
$g[v] = g[tmp]$
$G_j[v] = G_j[tmp]$ for $j = 0, 1, \ldots, J-1$
$\pi_g[v] = u$

Fig. 6 The relaxation procedure of subroutine Look_Ahead _Dijkstra in H_MCP.

Prefer_the_best (a, b)
if $(\forall j = 0, \cdots, J-1, G_j[a] + R_j[a] \leq C_j)$ **then** return (a)
if $(\forall j = 0, \cdots, J-1, G_j[b] + R_j[b] \leq C_j)$ **then** return (b)
if $(g[a] < g[b])$ **then** return (a)
return (b)

Fig. 7 The preference rule used in H_MCP.

Modified_H_MCOP$(G(V,E), s, t, c, w_j, C_j, j = 0, \cdots, J-1)$
$\mathbf{r} \leftarrow$ H_MCOP$(G(V,E), s, t, c, w_j, C_j, j = 0, \cdots, J-1)$
if ($\mathbf{r} \neq$ NULL) **then**
set $w_J(e) = c(e)$ for each link e
set $C_J = c(\mathbf{r}) - \epsilon$
repeat
$\mathbf{q} \leftarrow$ H_MCP$(G(V,E), s, t, w_j, C_j, j = 0, \cdots, J)$
/*MCP problem with $J+1$ constraints*/
if ($\mathbf{q} \neq$ NULL) **then**
$\mathbf{r} \leftarrow \mathbf{q}$
set $C_J = c(\mathbf{r}) - \epsilon$
until $\mathbf{q} =$ NULL
return $\mathbf{r}$

Fig. 8 The heuristic algorithm Modified_H_MCOP for the MCOP problem.

itive number such that no path between s and t has a cost in $[c(\mathbf{r}) - \epsilon, c(\mathbf{r})]$. H_MCP is hence employed to solve the new MCP problem. Obviously if a feasible solution $\mathbf{q}$ is found by H_MCP, $\mathbf{q}$ must have a lower cost than $\mathbf{r}$. To further search for better solutions, we can replace $\mathbf{r}$ by $\mathbf{q}$, update the upper bound C_J, and repeat this procedure until no feasible solution is found by H_MCP.

Modified_H_MCOP is an iterative algorithm. Other variants can be easily developed if one needs to firmly control the time complexity. For instance, we may restrict the number of times H_MCP runs. In the extreme case, we may let H_MCP run exactly once if a feasible solution is found by H_MCOP, and thus the Dijkstra's algorithm will be executed in total at most four times. Since Modified_H_MCOP (including the variants described above) starts from the solution of H_MCOP, and considering that H_MCP can obtain a feasible solution with a very high probability if such a path exists, Modified_H_MCOP can definitely surpass H_MCOP from the quality of solution standpoint. This will be demonstrated by the performance evaluation in the next Section.

5. Performance Evaluation

In this Section, the proposed algorithms are tested on a number of randomly generated networks with random link weights. The experiments are designed in a way very similar to the one used in [14] in order to achieve a relatively fair comparison between H_MCOP and our proposed modified heuristic algorithms. For this reason, Waxman's method [24] used in [14] is also employed here to generate network topologies, even though there exist some other methods for modeling real communication networks [6]. We assume that in the following experiments there are three constraints. The three weights corresponding to the constraints are uniformly distributed on different sets of intervals, while the cost is always uniformly distributed on $[1, 500]$. Three types of networks (50-, 100- and 200-node) are randomly generated. Given a network size and a set of intervals on which the link weights are distributed, a total number of 100,000 routing requests are generated as follows. First, 10 graphs are generated, then for each graph 10 instances of link weights are generated, and finally for each graph and each instance of link weights, 1000 routing requests are generated. To generate a routing request, a source and a destination are first randomly selected, then the shortest paths $\mathbf{p}_j$ w.r.t. $w_j, j = 0, 1, 2$ are computed via Dijkstra's algorithm, and we let the upper bound C_j of each constraint j be given by

$$C_j = \gamma w_j(\mathbf{p}_j), j = 0, 1, 2$$

where γ is a positive number called *constraint factor*.

For each routing request, both the exact algorithm E_MCOP and the heuristics H_MCOP and Modified_H_MCOP are employed to find a solution. Besides, two variants of Modified_H_MCOP, H_MCOP_MCP and H_MCOP_2MCP, are also tested. H_MCOP_MCP restricts H_MCP to be run exactly once if H_MCOP returns a feasible solution, while H_MCOP_2MCP restricts H_MCP to be run at most twice. For all of these heuristics, the parameter λ used for nonlinear relaxation in procedure Look_Ahead_Dijkstra is fixed to be 25.

For a specific algorithm, some of the following performance measures are computed based on its results for processing 100,000 routing requests on networks of a given size with link weights distributed on a specific set of intervals:

- *Success ratio* (SR), which is the percentage of routing requests that feasible solutions have been found. One should be aware that H_MCOP, Modified_H_MCOP and its variants have the same SR.
- *Success probability* (SP), which is the probability

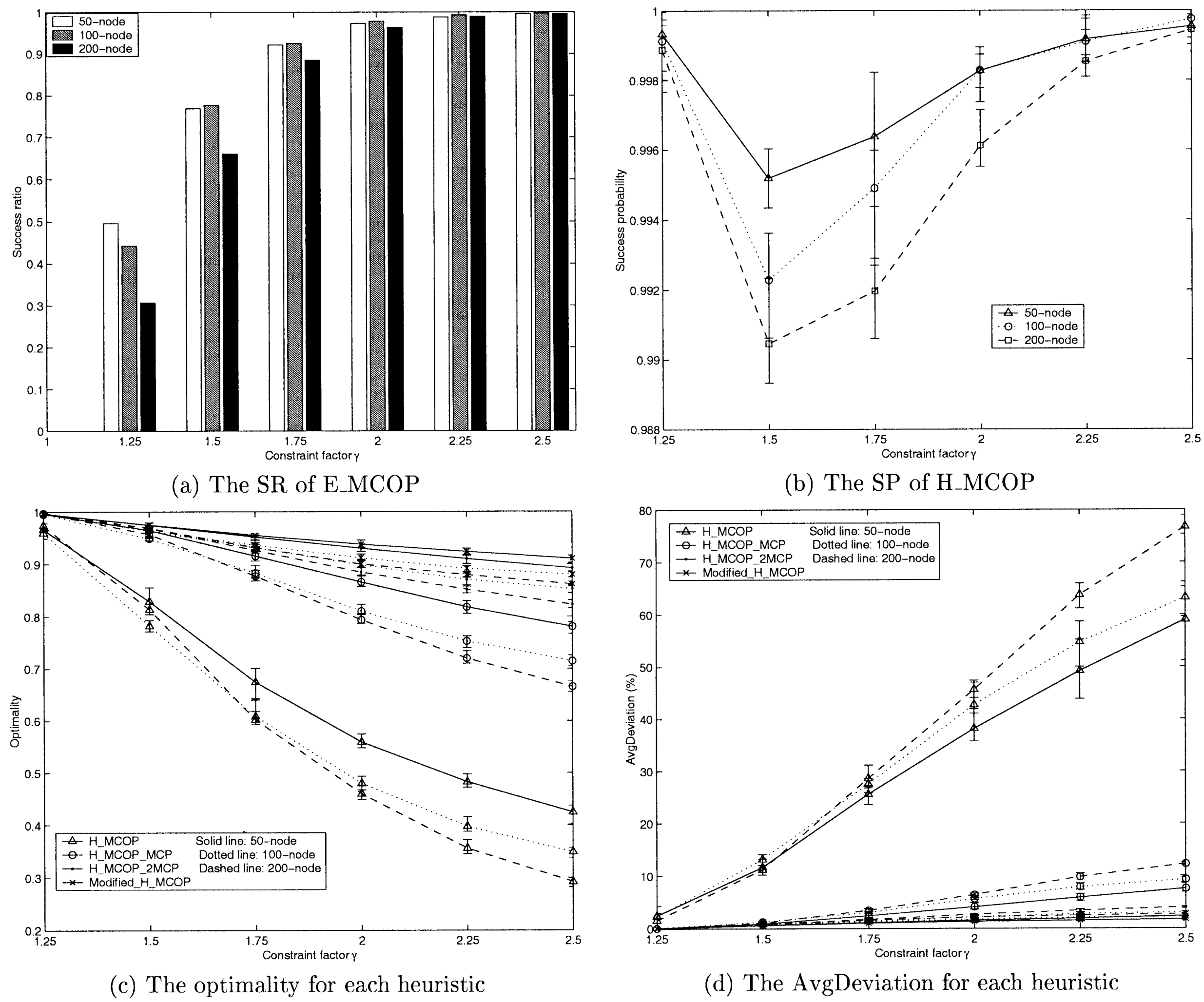


Fig. 9 Performance measures with various values for γ.

that a heuristic can successfully find a feasible path if there exists at least one. It can be calculated in terms of SR as follows

$$\text{SP of heuristic } \mathcal{H} = \frac{\text{SR of heuristic } \mathcal{H}}{\text{SR of algorithm E\_MCOP}}.$$

Similarly, H_MCOP, Modified_H_MCOP and its variants must have the same SP.

- *Optimality*, which is the probability that a heuristic can find the optimal solution if there exists at least one feasible path. If there are in total y routing requests for which there is at least one feasible path (found by E_MCOP), and among them there are x routing requests for which heuristic $\mathcal{H}$ has found the optimal solution, then the optimality of heuristic $\mathcal{H}$ is given by x/y.
- *Average cost deviation* (AvgDeviation), which is the average cost deviation in percentage between the feasible solution of a heuristic and the optimal solution.

In addition, other performance measures like the average number of executions of H_MCP in Modified_H_MCOP are also computed. For some performance measures, 95% confidence intervals are provided as shown in the following figures and tables.

5.1 Performance Analysis with Different Values of Constraint Factor

Assuming that the three link weights w_0, w_1 and w_2 are uniformly distributed on $[1, 200]$, $[100, 300]$ and $[200, 400]$, respectively, we first analyze the performance when the constraint factor γ takes six different values between 1.25 and 2.5. The performance measures for each case are shown in Fig. 9.

Figure 9(a) shows the average SR of the exact algorithm. According to our method for generating constraints stated above, we know that the larger the value of γ, the higher the probability that a feasible path exists. Thus, as shown in Fig. 9(a), the SR increases with the increase of γ. The SP of H_MCOP is shown

Table 1 Average/maximum number of executions of H_MCP with various values of γ.

γ	1.25	1.5	1.75	2	2.25	2.5
50-node	1.03±0 / 4	1.16±0.03 / 5	1.34±0.05 / 5	1.5±0.02 / 6	1.63±0.02 / 6	1.72±0.03 / 6
100-node	1.03±0 / 3	1.2±0.01 / 6	1.42±0.04 / 6	1.62±0.02 / 6	1.76±0.02 / 7	1.86±0.02 / 6
200-node	1.02±0 / 3	1.17±0.01 / 4	1.42±0.01 / 6	1.64±0.02 / 6	1.84±0.03 / 7	1.97±0.01 / 7

in Fig. 9(b), from which we may conclude that in general H_MCOP can find a feasible path with a very high probability if there exists one.

The optimality of heuristics H_MCOP, H_MCOP _MCP, H_MCOP_2MCP and Modified_H_MCOP is shown in Fig. 9(c), from which one may notice that when γ is given a moderate value (e.g., 2), H_MCOP_MCP can achieve a much higher optimality than H_MCOP. H_MCOP_2MCP and Modified_H_MCOP can further improve the optimality, but the improvements are less than the improvement between H_MCOP and H_MCOP_MCP. However, with the increase of γ, the difference between the optimality of these heuristics becomes more and more conspicuous, and the number of executions of H_MCP has to be increased to achieve better performance. Another observation is that for any heuristic the optimality decreases with the increase of γ. This is probably because the larger the value of γ, the more the feasible paths, and thus the harder the optimal solution can be found.

The average quality of solution can be reflected by AvgDeviation as shown in Fig. 9(d). Once again we can see that Modified_H_MCOP and its variants can significantly reduce the cost deviation. For instance, when $\gamma = 2.5$ and the network size is 50, the AvgDeviation for H_MCOP is about 60%, while it is below 10% for the modified algorithms.

When Modified_H_MCOP is employed to process a routing request for which there exists at least one feasible solution, one may wonder how long it takes the algorithm to converge. The average and maximum numbers of executions of H_MCP in such case are shown in Table 1. We notice that the average number of executions of H_MCP is between 1 and 2 when $\gamma = 2$. This indicates that in most cases the major improvement of the performance of H_MCOP is accomplished by the first run of H_MCP. This complies with our previous observation that H_MCOP_MCP can considerably outperform H_MCOP, while H_MCOP_2MCP and Modified_H_MCOP can only improve the optimality by a little more than H_MCOP_MCP. However, with the increase of γ, the average number of executions of H_MCP also increases and it is possible that H_MCOP_2MCP makes a considerable improvement over H_MCOP_MCP, or Modified_H_MCOP over H_MCOP_2MCP. Therefore one should carefully choose the number of executions of H_MCP in order to make the best tradeoff between the time complexity and the performance.

Table 2 Three sets of link-weight intervals.

Set number	1	2	3
w_0	$[1, 300]$	$[1, 100]$	$[1, 100]$
w_1	$[1, 300]$	$[100, 200]$	$[1, 1000]$
w_2	$[1, 300]$	$[200, 300]$	$[1, 10000]$

Table 3 Average/maximum number of executions of H_MCP with various sets of link-weight intervals.

Set number	1	2	3
50-node	1.51±0.02/7	1.7±0.02/6	1.52±0.03/6
100-node	1.62±0.03/7	1.82±0.02/6	1.63±0.02/7
200-node	1.68±0.02/7	1.96±0.01/8	1.68±0.02/7

5.2 Performance Analysis with Different Sets of Link-Weight Intervals

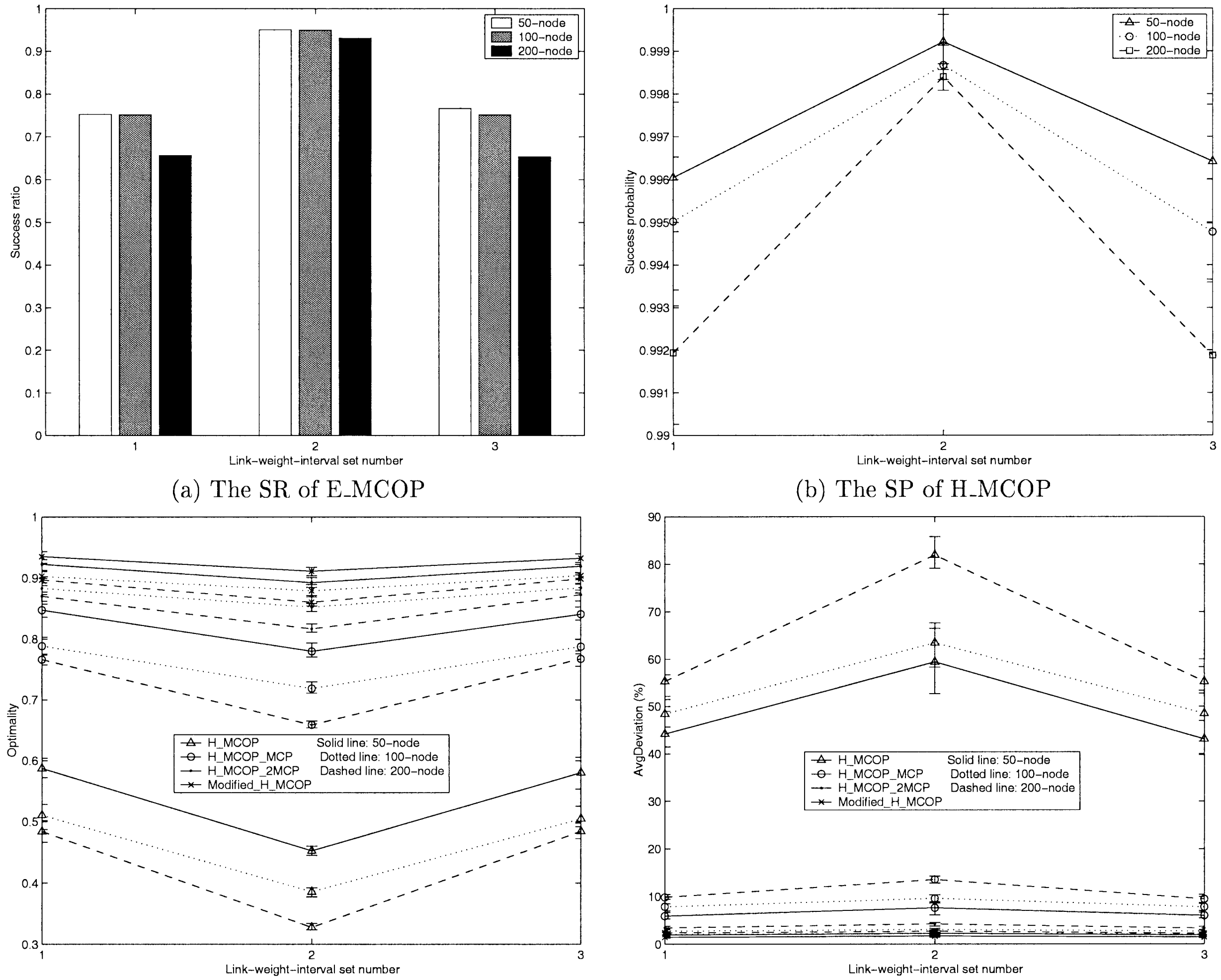
Now we investigate the performance of the heuristics when link weights are distributed on three different sets of intervals, which are enumerated in Table 2. In this case we let the constraint factor γ for each routing request be randomly selected from $[1.2, 4]$. Correspondingly, the performance measures are shown in Fig. 10 and Table 3.

From Fig. 10 we can see that on one hand the heuristics can still achieve a very high SP, while on the other hand Modified_H_MCOP and the variants can still significantly outperform H_MCOP in terms of the optimality or the average cost deviation.

While we can see the link-weight intervals do affect the performance of these algorithms, they do not have substantial impacts. We notice that with the second set of link-weight intervals, it is more difficult for the heuristics to find the optimal solution in comparison with the cases with other two sets of link-weight intervals. This is due to the same reason as mentioned above, i.e., on average there are more feasible paths with the second set of link-weight intervals, as can be seen from the SR in Fig. 10(a).

5.3 Other Relevant Statistics

Although the performance of the exact algorithms is not our major concern in this paper, some statistical results are also provided for future reference. Corresponding to the experiments in Sect. 5.1, Table 4 shows the average/maximum number of shortest paths checked by E_MCP/E_MCOP before a feasible/optimal solution is found or a conclusion is drawn that no feasible path is available, while these performance measures corresponding to the experiments in Sect. 5.2 are shown



(a) The SR of E_MCOP (b) The SP of H_MCOP

(c) The optimality for each heuristic (d) The AvgDeviation for each heuristic

Fig. 10 Performance measures with various sets of link-weight intervals.

Table 4 Average/maximum number of paths checked by E_MCP/E_MCOP with various values of γ.

γ	50-node		100-node		200-node	
	E_MCP	E_MCOP	E_MCP	E_MCOP	E_MCP	E_MCOP
1.25	5.78±0.54 / 126	7.73±0.67 / 126	8.38±2.03 / 172	10.61±2.02 / 172	15.53±1.98 / 246	17.85±1.94 / 246
1.5	4.82±1.57 / 480	7.88±1.45 / 482	6.56±1.68 / 529	10.33±1.72 / 532	15.68±2.85 / 2085	20.16±2.98 / 2102
1.75	3.31±1.23 / 387	6.94±1.49 / 409	5.08±3.46 / 1690	9.63±3.7 / 1690	11.97±1.98 / 6139	17.69±2.05 / 6139
2	2.44±0.57 / 511	6.72±0.53 / 523	3.1±0.54 / 1557	8.6±0.77 / 1620	9.08±2.77 / 4454	15.95±3.12 / 4454
2.25	1.91±0.33 / 250	7.05±0.57 / 426	2.1±0.52 / 557	9.24±0.69 / 2462	5.47±1.8 / 4825	14.48±1.34 / 4830
2.5	1.57±0.27 / 421	8.07±0.58 / 1041	1.65±0.27 / 323	11.27±0.94 / 1097	3.47±1.58 / 8459	16.01±1.3 / 8500

Table 5 Average/maximum number of paths checked by E_MCP/E_MCOP with various sets of link-weight intervals.

Set #	50-node		100-node		200-node	
	E_MCP	E_MCOP	E_MCP	E_MCOP	E_MCP	E_MCOP
1	1.79±0.12 / 33	14.3±3.33 / 15916	1.82±0.16 / 42	37.39±34.87 / 202028	2.28±0.15 / 45	30.4±4.66 / 30366
2	1.77±0.18 / 191	12.45±1.11 / 6044	2.04±0.26 / 510	23.47±7.61 / 67151	3.4±0.65 / 2571	43.69±8.59 / 105591
3	1.73±0.2 / 27	16.19±6.1 / 8062	1.83±0.17 / 41	28.99±7.28 / 33554	2.31±0.1 / 77	29.93±9.84 / 49791

in Table 5.

Observing these data, we can see that for these experiments, in most cases (note the 95% confidence intervals) E_MCP only needs to check up to 20 paths, while E_MCOP 80 paths. Even though these numbers are expected to increase with the increase of the net-

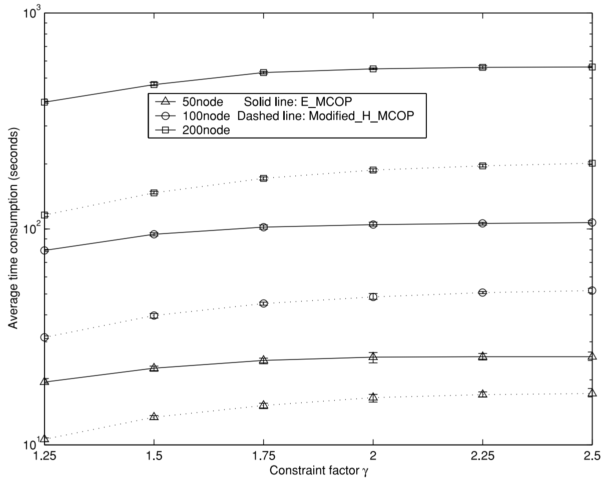


Fig. 11 The average time consumption of E_MCOP and Modified_H_MCOP for processing 10,000 routing requests.

work size, it will not be very time consuming (on average) to find the optimal solutions even for a large number of routing requests on a network of a moderate size. Therefore, they may be very useful for evaluating other heuristics. However, we should also notice that the maximum number of paths checked by E_MCP or H_MCOP in some cases is extremely large. Therefore in the worst case the exact algorithms are still very time consuming and thus cannot be used in real-time applications.

The actual computing times of the exact algorithm E_MCOP and the heuristic algorithm Modified_H_MCOP are also obtained when these algorithms run on a Dell Pentium III computer. Figure 11 shows the average time the two algorithms need, respectively, to process 10,000 routing requests in the experiments in Sect. 5.1. One may notice that the computing time increases with the value of γ. This complies with our previous analysis as well as the statistical results in Table 1. One may also see that the average computing time of Modified_H_MCOP is much less than that of the exact algorithm. The computing time of other algorithms proposed in this paper can be estimated from the data shown in Fig. 11. For instance, under the same condition, the variants of Modified_H_MCOP should take less time than Modified_H_MCOP. Also, for a given routing request, on average E_MCP needs much less computing time to find a feasible path than E_MCOP needs to find the optimal solution. It should be pointed out that with different implementations of the KSP algorithm used in the exact algorithms, the computing time of E_MCOP (or E_MCP) may be different. Our implementation is based on the algorithm proposed in [5]. Since it is a not loop free algorithm, we have made slight modifications so that it only returns loopless paths.

6. Conclusions

As an important problem in QoS unicast routing, the problem of finding the optimal path subject to multiple constraints is studied in this paper. Both exact and approximate algorithms have been investigated for solving this problem. The proposed exact algorithm first constructs an aggregate weight for each link and then uses a K-shortest-path algorithm to search for the optimal solution. While in the worst case the exact algorithm has to check a large number of paths to identify the optimal solution or draw a conclusion, for the majority of experiments the number of paths to be checked is very small. Therefore, on one hand, the exact algorithm is not suitable for real-time applications, while on the other hand it is very useful to evaluate the performance of other heuristic algorithms.

After making comparisons with the exact algorithm, we find that the heuristic algorithm H_MCOP [14] can find a feasible path with a very high probability if there exists one. However, its optimality or average cost deviation from the optimal solution is not good enough. For this reason, we propose some modified heuristics based on H_MCOP that can significantly improve the optimality with slightly increased time complexity. This is demonstrated by a large quantity of simulations on various networks with various link-weight intervals. A few variants of the modified heuristic can be developed to achieve the best tradeoff between the time complexity and the performance, enabling one to choose the most appropriate heuristic in a given situation.

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Appendix: Heuristic Algorithm H_MCOP

Figure A·1 shows the heuristic algorithm H_MCOP proposed in [14], which intends to find a path that satisfies a total number of J additive constraints. A cost function is defined in H_MCOP as follows:

$$g_\lambda(\mathbf{p}) = \left(\frac{w_0(\mathbf{p})}{C_0}\right)^\lambda + \left(\frac{w_1(\mathbf{p})}{C_1}\right)^\lambda + \cdots + \left(\frac{w_{J-1}(\mathbf{p})}{C_{J-1}}\right)^\lambda$$

H_MCOP$(G(V,E), s, t, w_j, C_j, j = 0, 1, \ldots, J-1)$
  Reverse_Dijkstra $(G(V,E), t)$
  if $(r[s] > J)$ **then**
    return failure /* no feasible path available */
  Look_Ahead_Dijkstra$(G(V,E), s)$
  if $(G_j[t] \le C_j, \forall j = 0, 1, \ldots, J-1)$ **then**
    return the path
  return failure

Fig. A·1 The heuristic algorithm H_MCOP for multi-constrained optimal path (MCOP) problem.

Reverse_Dijkstra_Relax(u, v)
  if $r[u] > \sum_{j=0}^{J-1} \frac{R_j[v]+w_j(u,v)}{C_j}$ **then**
    $r[u] = \sum_{j=0}^{J-1} \frac{R_j[v]+w_j(u,v)}{C_j}$
    $R_j[u] = R_j[v] + w_j(u,v)$
    $\pi_r[v] = u$

Fig. A·2 The relaxation procedure of subroutine Reverse_Dijkstra.

where $\lambda \ge 1$. The inputs of this algorithm include: a directed graph $G(V,E)$ in which each link is associated with a primary cost c and J weights $w_j, j = 0, 1, \ldots, J-1$, an upper bound C_j for each constraint, a source s and a destination t. The algorithm maintains for each node u the following labels: $r[u], R_j[u], \pi_r[u], g[u], G_j[u], \pi_g[u]$, and $c[u]$,$j = 0, 1, \ldots, J-1$. Label $r[u]$ denotes the cost of the shortest path from u to t w.r.t. the cost function g_1, i.e., g_λ with $\lambda = 1$. Labels $R_j[u]$, $j = 0, 1, \cdots, J-1$ represent individually the accumulated link weights along that path. The predecessor of u along the path is stored in $\pi_r[u]$. Label $g[u]$ represents the cost of a complete path from s to t via node u w.r.t. cost function g_λ $(\lambda > 1)$. Labels $G_j[u]$, $j = 0, 1, \cdots, J-1$ and $c[u]$ represent individually the accumulated link weights and the primary cost from s to u along the path. The predecessor of u along the path is stored in $\pi_g[u]$.

Subroutine Reverse_Dijkstra runs Dijkstra's algorithm in a reverse direction with node t being the input value. The relaxation procedure used in this subroutine is shown in Fig. A·2. One should notice that notation (u, v) denotes the link from u to v. As a result, Reverse_Dijkstra can find the shortest path from every node u to t w.r.t. the cost function $g_1(\cdot)$.

After the execution of Reverse_Dijkstra, H_MCOP checks the condition $r[s] > J$ to judge if a feasible path exists based on a theorem proved in [14]. In case that the condition is false, H_MCOP precedes to call subroutine Look_Ahead_Dijkstra, which runs the standard Dijkstra's algorithm with a modified relaxation procedure described in Fig. A·3. This subroutine starts with s and choose the next node based on the preference rule in Fig. A·4. This rule will choose the next node that can lead to the primary cost being minimized if there exists a foreseen feasible path between s and t, and otherwise the cost function $g_\lambda(\cdot)$ being minimized.

Look_Ahead_Dijkstra_Relax(u, v)

Let tmp be a temporary node
$c[tmp] = c[u] + c(u, v)$
if $\lambda < \infty$ **then**
$g[tmp] = \sum_{j=0}^{J-1} \left(\frac{G_j[u]+w_j(u,v)+R_j[v]}{C_j} \right)^{\lambda}$
if $\lambda = \infty$ **then**
$g[tmp] = \max \left\{ \frac{G_j[u]+w_j(u,v)+R_j[v]}{C_j} | 0 \leq j \leq J-1 \right\}$
$G_j[tmp] = G_j[u] + w_j(u, v)$ for $j = 0, 1, \ldots, J-1$
$R_j[tmp] = R_j[v]$ for $j = 0, 1, \ldots, J-1$
if (Prefer_the_best$(tmp, v) = tmp$) **then**
$c[v] = c[tmp]$
$g[v] = g[tmp]$
$G_j[v] = G_j[tmp]$ for $j = 0, 1, \ldots, J-1$
$\pi_g[v] = u$

Fig. A· 3 The relaxation procedure of subroutine Look_Ahead _Dijkstra.

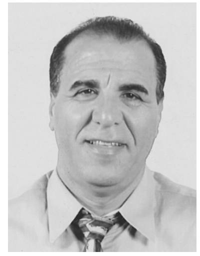
Prefer_the_best (a, b)

1 **if** $c[a] < c[b]$ and $(\forall j, G_j[a] + R_j[a] \leq C_j)$ **then** return (a)
2 **if** $c[a] > c[b]$ and $(\forall j, G_j[b] + R_j[b] \leq C_j)$ **then** return (b)
3 **if** $(g[a] < g[b])$ **then** return (a)
4 return (b)

Fig. A· 4 The preference rule used in H_MCOP.

After the execution of Look_Ahead_Dijkstra, H_MCOP will return the feasible path recovered from labels $\pi_g[\cdot]$ if such a path exists.

Gang Feng received his B.E. and M.E. from University of Electronic Science and Technology of China in 1992 and 1995 respectively, and Dr.Eng. from Beijing University of Posts and Telecommunications in 1998, all in electrical engineering. Between 1998 and 2001, he was studying in University of Miami, where he was granted Ph.D. in December 2001. Since July 2001, he has been working in Telecommunications & Information Technology Institute at Florida International University as a Research Associate. His major research interests include routing in next generation broadband wired/wireless networks, optical networking, combinatorial optimization, neural networks and evolutionary computation.



Kia Makki received his Ph.D. in computer science from the University of California, Davis, and two M.S. degrees from The Ohio State University and West Coast University. He is currently a tenure professor with an endowed chair professorship at Telecommunications & Information Technology Institute in Florida International University. His technical interests include wireless and optical telecommunications. He has over hundred refereed publications and his work has been extensively cited in books and papers. He has been Associate Editor, Editorial Board Member, Guest Editor of several International Journals including Editorial Board Member of several telecommunication journals. He has been involved in numerous conferences as a Steering Committee Member, General Chair, Program Chair, Program Vice-Chair and Program Committee Member.

Niki Pissinou received her Ph.D. in Computer Science from the University of Southern California, her M.S. in Computer Science from the University of California at Riverside, and her B.S.I.S.E. in Industrial and Systems Engineering from The Ohio State University. She is currently as a tenured professor and the director of the Telecommunication & Information Technology Institute at FIU. She has published over hundred refereed publications and has received best paper awards. She has also coedited seven volumes and is the author of an upcoming book on wireless Internet computing. She has also served as a steering committee, general chair and program committee member of over hundred programs and organizational committees of IEEE and ACM sponsored technical conferences. She has also refereed for a wide variety of conferences and journals, and has served as a reviewer on NSF and NASA panel reviews including the KDI and ITR panels. She has been the editor of eight journals including IEEE TKDE and guest editor of seven journals. She has received seven achievements awards and an invited speaker and keynote speaker of conferences.

Christos Douligeris was born in Melissopetra, Arcadia, Greece. He received his Diploma in Electrical Engineering from the National Technical University of Athens in 1984 and the M.S., M.Phil. and Ph.D. degrees from Columbia University in 1985, 1987, 1990 respectively. He has held positions with the Department of Electrical and Computer Engineering at the University of Miami, where he reached the rank of associate professor and was the associate director for engineering of the Ocean Pollution Research Center. He has also held visiting positions at the National Technical Univ. of Athens and the Univ. of Piraeus, Greece. He has served in technical program committees of several conferences. His main technical interests lie in the areas of performance evaluation of high speed networks, neurocomputing in networking, resource allocation in wireless networks and information management, risk assessment and evaluation for emergency response operations. He is an editor of the IEEE Communications Letters and the IEEE Network, a technical editor of the IEEE Communications Magazine, a technical editor of the IEEE Communications Magazine Interactive, an associate editor of Computer Networks, a senior member of IEEE, and a member of INFORMS (formerly ORSA) and the Technical Chamber of Greece. During the academic year 1999–2000 he worked in the National Greek Agricultural Foundation as a Senior Fulbright Scholar.