Division Rule

There are $n/d$ ways to do a task if it can done using a procedure that can be carried out in $n$ ways, and for every way $w$, exactly $d$ of the $n$ correspond to way $w$.

(A rule to ignore “unimportant” differences when counting things.)

Example 1 (revisited): There are 15 different sandwich and drink combinations for a lunch. If the choice of a drink does not matter, and there are three drink choices, what is the number of ways to pick a lunch?

Three possibilities for a sandwich:
- Coffee + a sandwich
- Soda + a sandwich
- Juice + a sandwich

Example 12: How many ways are there to seat 5 people around a round table?
Suppose we consider two seatings the same when everybody has the same left and right neighbor. How many seatings are possible in this case?

**Example 13:** Suppose we are seating 5 people in a line. If the left-to-right order is important, then there are 5! ways form a line.

Suppose that only the placements that result in distinct adjacencies of people are considered:
Problem 22, Section 6.1
Consider all positive integers less than 1000.

(a) Calculate the number integers divisible by 7:

(c) Calculate the number of integers divisible by 7 and 11:

(b) Number of integers divisible by 7 but not by 11:

(d) Number of integers divisible by 7 or 11:

(e) Number of integers divisible by exactly one of 7 or 11:

(f) Divisible by neither 7 nor 11:
**Problem 14** How many bit strings of length $n$, where $n$ is a positive integer, start and end with 1s?

**Problem 25** How many strings of three decimal digits

(a) do not contain the same digit three times?

(b) begin with an odd digit?

(c) have exactly 2 digits that are 4s?
Problem 44 How many ways can four out of ten people be seated around a circular table if multiple seating arrangements that result in the same neighbors counted only once?

Problem 45 Count the number of ways six people are seated around a round table. Multiple seating arrangements that result in the same neighbors regardless of left or right direction are considered the same and counted only once.
Problem 33 Consider strings of eight English letters.
(d) Count the number of strings that start with a vowel, letters cannot be repeated

(g) Count the number of strings that start with x and contain at least one vowel; letters can be repeated

Problem 47 Consider the problem of forming a row of 6 people, including a bride and a groom, for picture taking. Count the number of ways a row can formed if
(a) the bride and groom must be next to each other.

(b) the bride and groom must not be next to each other.
(c) the bride is to the left of the groom.