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Constructions with Pentacubes–2

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In this paper we wish to give five more pentacube constructions similar to those in Reference 1. As before, we work with the twenty-eight pieces obtained by connecting five unit cubes together on their faces, not counting the "I" pentacube. [We use the terminology of Reference 2 (p. 23) for the twelve flat pentacubes. See also References 3 (p. 180) and 2 for more discussion of pentacubes.] Each of our illustrations shows the various levels, where the bottom level is numbered 1, and a dot represents a cube attached above.

Figure 1 gives four modules from which one can assemble triple-sized models of eleven of the pentacubes, with one pentacube left over. These eleven include ten that were not obtained previously [1], so that the two constructions together yield models of every pentacube except the "Y." (In some of the constructions, the "N" pentacube must be shifted and the "F" and "W" pentacubes must be interchanged.)

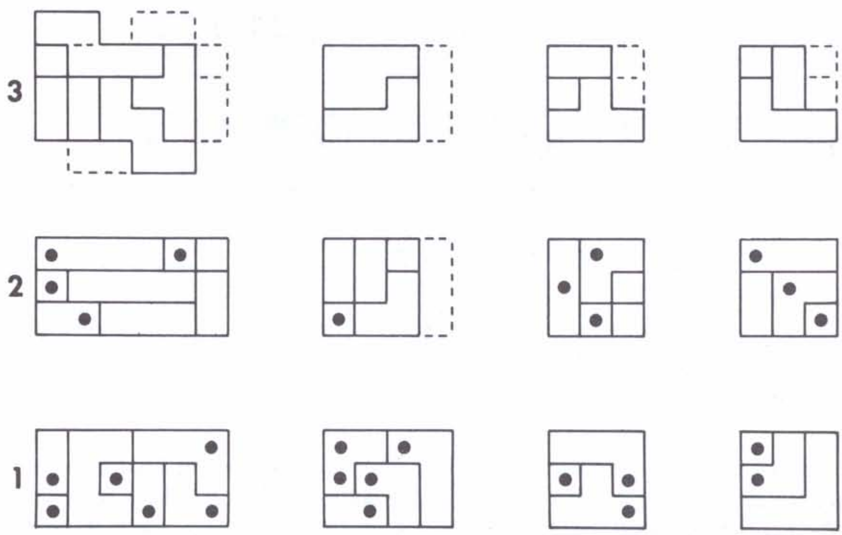

Figure 1

Figure 2 shows the simultaneous construction of double-sized models of three pentacubes ("L," "V," and "P") with those three pentacubes and one other ("F") left over.

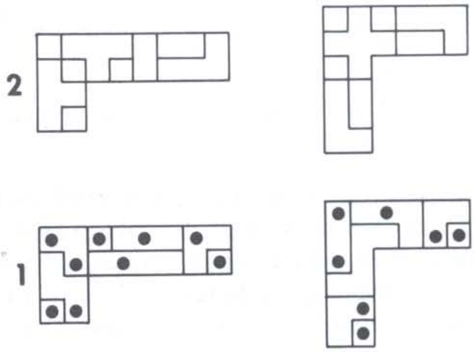

Figure 2

Figure 3 illustrates the construction of seven cylinders five units high. As noted [1], it is impossible to construct eight or more such cylinders (this would involve constructing four or more cylinders with three pentacubes each), so in this sense Figure 3 cannot be bettered. Figure 3 also illustrates cross sections of a few of the possible cylinders five cubes high which can be constructed from these modules.

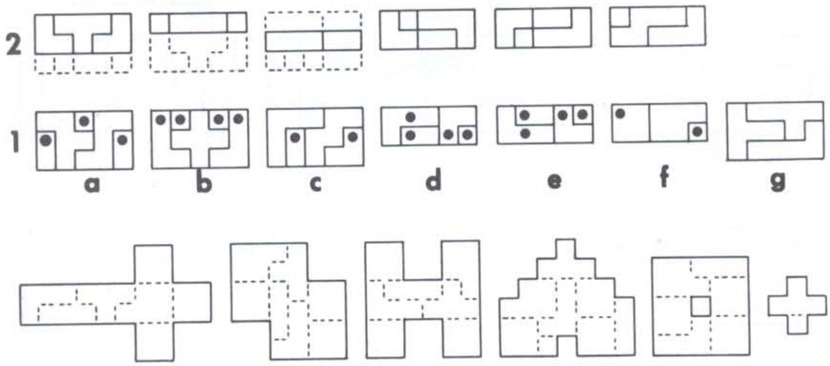

Figure 3

Let us count the number of distinct ways the modules in Figure 3 can be used to construct a 4 X 5 X 7 solid, where solutions differing by a rotation or reflection of the solid are not considered distinct. Analyzing, as in Reference 1, there are 17 "basic types" of solutions (ways of arranging cross sections of the modules into a 4 X 7 rectangle, not counting rotations or reflections), and

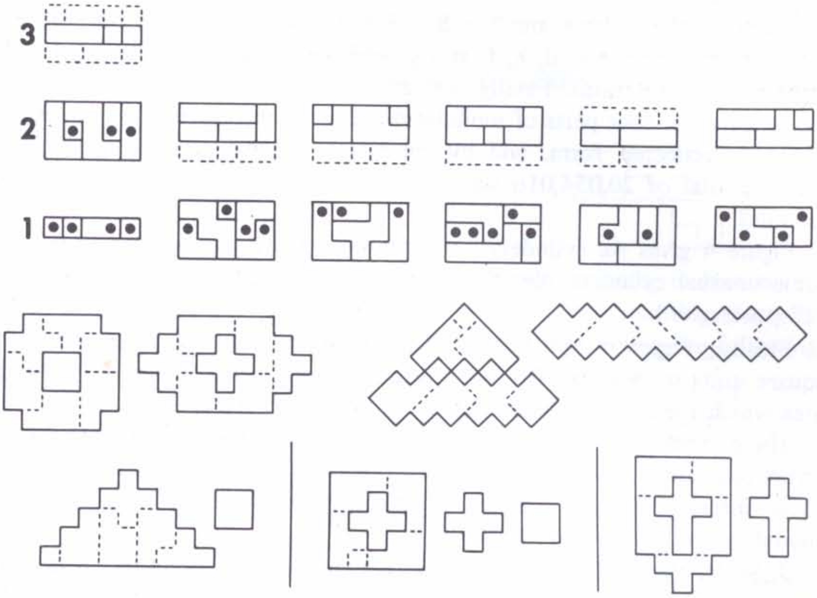

Figure 4

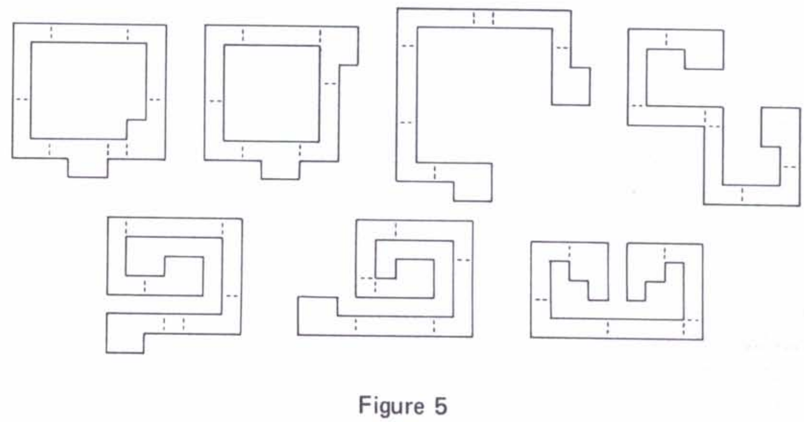

Figure 5

for each of these there are 2 · 8 · 8 · 8 · 4 · 6 solutions resulting from rotations of modules c, d, e, f, and g, and permutations of d, e, and f. This gives 417,792 solutions. Finally, we can multiply by a factor of 3 · 8 · 2 by rearranging the four parts of modules d and e, by reconstructing modules a, c, and f in reflected form, and by completely reconstructing module f. This yields a total of 20,054,016 ways to construct a 4 X 5 X 7 solid from these modules.

Figure 4 gives six cylinders five units high which can be used to construct some unusual cylinders, also shown in Figure 4. (A few of these include the "I" pentacube.)

Finally, Figure 5 shows six cylinders which will enclose an area of 29 square units with a wall five units high, as shown. This represents the largest area which the author has managed to enclose up to now.

These examples illustrate the fantastic possibilities for pentacube constructions. The reader with a set of pentacubes can find many other, perhaps more interesting, constructions. Most constructions seem easier to do in modular form than to do directly.

J. M. M. Verbakel of Philips Research Laboratories, Eindhoven, Netherlands has produced a very interesting construction of the type in Figure 3 in which all seven modules have different cross sections.

References

1. N. R. Wagner, "Constructions with Pentacubes," *J. Recreational Math.*, 5(4), October 1972, pp. 266-268.
2. Solomon W. Golomb, *Polominoes*, Charles Scribner's Sons, New York, 1965.
3. Martin Gardner, "Mathematical Games," *Scientific American*, 227(3), September 1972, pp. 176-182.