

Discrete Mathematical Structures CS 2233 Lecture Eleven

Prof. William Winsborough
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Business

- **Homework 6**, due Thursday 3/5
 - 2.4: 2, 32, 36
 - 3.1: 2, 4, 12, 18, 24
- Read Sections 2.4 and 3.1
- Questions???

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Bijections and Cardinality

- Sets A and B are *equinumerous* (meaning they have the same cardinality) iff there is a one-to-one correspondence between them
 - Notice that this defines a binary relation over sets
 - Contains the ordered pairs of sets between which there exists a one-to-one correspondence
- Pretty clear for finite sets

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Countability

- *Recall*: Sets A and B are *equinumerous* (meaning they have the same cardinality) iff there is a one-to-one correspondence between them
- **Definition**
 - A set A is *countable* if it is finite or it is equinumerous to \mathbb{N}
 - Otherwise, A is *uncountable*

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Sequences

- **Definition**
 - A *sequence* is a function from \mathbb{N} or \mathbb{Z}^+ to a given set S
 - We use a_n to denote the image of n
 - We use $\{a_n\}$ to denote the whole sequence
 - Less formally, we sometimes denote it by $\{a_0, a_1, a_2, a_3, \dots\}$
 - If the function is onto we say $\{a_n\}$ enumerates S

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Countability and Enumeration

- **Theorem**: S is countable if and only if there exists a sequence that enumerates S
- **Proof**
 - Only if: If there is a bijection between \mathbb{N} and S , it is a sequence that enumerates S
 - If: Given a sequence that enumerates S , either S is finite or dropping repeated values from the sequence yields a bijection between \mathbb{N} and S
- **Theorem**: \mathbb{Q}^+ , the set of positive rationals, is countable

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Uncountability of the Reals

- Theorem
 - \mathbb{R} , the set of real numbers, is uncountable
- Proof
 - Uses Georg Cantor's *diagonalization argument*
 - Outline
 - Assume for contradiction that there is a one-to-one correspondence, f , between \mathbb{N} and the real interval $[0,1]$
 - Use f to construct a real in $[0,1]$ that has no preimage under f
 - Idea: for each decimal place, n , in the representation of the constructed value, choose a decimal different from the n^{th} place of $f(n)$
 - The fact that the constructed value differs from each value assumed by f shows that f is not onto, giving the desired contradiction