Discrete Mathematical Structures
CS 2233 Lecture Seventeen

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Business

- Recall:
  - **Homework 8**, due Thursday 3/26
    - 4.1: 4, 6
    - 4.2: 4
  - Read sections 4.1, 4.2 and 4.3
  - Midterm II is on 4/2
- Tuesday 3/31 is review day
- IDEA survey will be administered Tuesday 3/31
  - Juan and Justin: Please collect the survey from Erika
- Mock Exam is handed out today
  - We will spend some time this Thursday (3/26) on review

Recursive Definition (Sec. 4.3)

- A *recursive definition* (also called an *inductive definition*) of a function f over N is given by specifying the value of the function in each of two cases:
  - The base case: f(0) is defined; more generally f(i) may be defined for all i less or equal to some $k \in N$
  - The recursive case: f(n+1) is defined in terms of f(n), f(n-1), …, f(0)
- Observe that f is a sequence

Examples

- Factorial F(n) = n!
  - F(0) = 1
  - F(n+1) = (n+1) F(n)
  - Defines the sequence {0!, 1!, 2!, …}
- Exponentiation
  - $a^0 = 1$
  - $a^{n+1} = a \cdot a^n$
- Σ: Sum of first n elements of a sequence $\{a_k\}$
  - $\sum_{0 \le i \le 0} a_i = 0$
  - $\sum_{0 \le i \le n+1} a_i = \sum_{0 \le i \le n} a_i + a_{n+1}$

Fibonacci Numbers

- The *Fibonacci numbers*, $f_0, f_1, f_2, \ldots$, are defined by:
  - $f_0 = 0$
  - $f_1 = 1$
  - $f_n = f_{n-1} + f_{n-2}$ for n > 1
- {0,1,1, 2, 3, 5, 8, …}

Inductive Proof about Recursively Defined Fibonacci Sequence

- Theorem: $\sum_{1 \le i \le n} f_{2i-1} = f_{2n}$, for n>0
- Basis (n=1): $\sum_{1 \le i \le 1} f_{2i-1} = f_1 = 0 + f_1 = f_0 + f_1 = f_2$
- Inductive Step (n+1):

$\sum_{1 \le i \le n+1} f_{2i-1} = \sum_{1 \le i \le n} f_{2i-1} + f_{2(n+1)-1}$

$= f_{2n} + f_{2n+1}$ by the induction hypothesis

$= f_{2n+2} = f_{2(n+1)}$