

## Discrete Mathematical Structures CS 2233 Lecture Thirteen

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## Business

- **Homework 7**, due Thursday 3/19
  - 3.2: 2, 4, 6, 8, 14, 20, 22
  - 3.3: 2
- Read Sections 3.2 and 3.3
- **Recall:**
  - Homework 6, due Thursday 3/5
    - 2.4: 2, 32, 36
    - 3.1: 2, 4, 12, 18, 24
  - Read Sections 2.4 and 3.1
- Questions???

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## Binary Search

```
proc binary search(x: int; a1, a2, ..., an: increasing ints)
i:=1 {left end of search interval}
j:=n {right end of search interval}
while i<j begin
  m:= ⌊(i+j)/2⌋
  if x > am then i:=m+1 else j:= m
end
if x=ai then location := i else location := 0
```

- How many comparisons are performed?
  - This will be studied in Section 3.3

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## Bubble Sort

```
proc bubble sort(a1,a2,...,an)
for i := 1 to n-1
  for j := 1 to n-i
    if aj > aj+1 then interchange aj and aj+1
  {a1,a2,...,an is in increasing order}
```

- How many comparisons are performed?
  - $\sum_{i=1}^{n-1} (n-i)$  (We evaluated this precisely on the board, obtaining the value  $n^2/2 - n/2$ . This motivates studying the growth of functions and asymptotic complexity)

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## Growth of Functions

- Big-O Notation
- Def: Let  $f, g : Z \rightarrow R$ . We write  $f(x) = O(g(x))$  if there are constants  $C$  and  $k$  such that  $|f(x)| \leq C|g(x)|$  for all  $x \geq k$ 
  - We say “ $f(x)$  is big-oh of  $g(x)$ ”
  - Technically  $O(g(x))$  is a set of functions and writing  $f(x) = O(g(x))$  really mean  $f(x) \in O(g(x))$
- When discussing positive-valued functions, we can drop the  $|\cdot|$

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## Example

- Let  $f(x) = x^2 + 2x + 1$  and  $g(x) = x^2$
- $f(x)$  is  $O(g(x))$ 
  - $k = 1$  and  $C = 4$  (witnesses)
  - $k = 2$  and  $C = 3$
- $g(x)$  is  $O(f(x))$ 
  - $k = 1$  and  $C = 1$

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