Discrete Mathematical Structures

CS 2233 Lecture Twenty-Two

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Business

- Turn in Homework 10
- Reading:
  - 9.1
  - 9.2 (pp. 597-600)
  - 9.3 (pp. 611-614)
- Homework 11 due Tuesday 4/28
  - 8.1: 4, 6, 14
    - Add 4e: a is at least as old as b
  - 8.4: 16
  - 8.5: 8

Properties of Binary Relations over a Single Set

- Let $r \subseteq A \times A$ be a binary relation over A
  - Reflexivity: r is *reflexive* if for all $a \in A$, $(a, a) \in r$ holds (i.e., is true)
  - Symmetry: r is *symmetric* if for all $a_1, a_2 \in A$, $(a_1, a_2) \in r \rightarrow (a_2, a_1) \in r$
  - Antisymmetry: r is *antisymmetric* if for all $a_1, a_2 \in A$, $(a_1, a_2) \in r \land (a_2, a_1) \in r \rightarrow a_1 = a_2$
  - Transitivity: r is *transitive* if for all $a_1, a_2, a_3 \in A$, $(a_1, a_2) \in r \land (a_2, a_3) \in r \rightarrow (a_1, a_3) \in r$

Equivalence Classes and Partitions

- Let $r \subseteq A \times A$ be an equivalence relation over A
  - The equivalence class of any $a \in A$ is given by $[a]_r = \{a' \mid a' \in A \land (a, a') \in r\}$
  - So we have $[a]_r \subseteq A$
  - Theorem: for all $a_1, a_2 \in A$, either $[a_1]_r = [a_2]_r$ or $[a_1]_r \cap [a_2]_r = \emptyset$
- A set $\{A_1, A_2 \ldots, A_n\}$ of subsets of A forms a partition if
  - For each i and j, $(A_i = A_j \lor A_i \cap A_j = \emptyset)$, and
  - $A_1 \cup A_2 \cup \ldots \cup A_n = A$
- Theorem: if $r \subseteq A \times A$ is an equivalence relation, the set of equivalence classes of elements of A form defines a *partition* of A
  - The set of equivalence classes is $\{[a]_r \mid a \in A\}$