Discrete Mathematical Structures
CS 3233 Lecture Sixteen

Prof. William Winsborough

October 10, 2005

Business

- Assignment 5 due Thursday, October 13, 2pm, at Dr. Winsborough's office
  - Section 2.1
    - 12, 24, 26, 42
    - In exercise 24, you may assume that the following set-iterator operation exists and is available for your use:
      - For each <variable> in <set> do <statement block>
- Midterm will be returned later this week
- Any problems while I was gone?

Bubble Sort

proc bubble sort($a_1, a_2, \ldots, a_n$)

for i := 1 to n-1

  for j := 1 to n-i

  if $a_j > a_{j+1}$ then interchange a_j and a_{j+1}

{$a_1, a_2, \ldots, a_n$ is in increasing order}

- How many comparisons are performed?

Insertion Sort

proc insertion sort($a_1, a_2, \ldots, a_n$: reals with $n \geq 2$)
for $j := 2$ **to** n
begin
  $i := 1$
  while $a_j > a_i$
    $i := i+1$
  $m := a_j$
  for $k := 0$ to $j - i - 1$
    $a_{j-k} := a_{j-k-1}$
  $a_i := m$
end {$a_1, a_2, \ldots, a_n$ is in increasing order}

Growth of Functions

- Big-O Notation
- Def: Let f, g : Z → R. We say f(x) is O(g(x)) if there are constants C and k such that $|f(x)| \leq C|g(x)|$ for all $x > k$
  - We say "f(x) is big-oh of g(x)"
- When discussing positive-valued functions, we can drop the $|\cdot|$

Example

- Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2$
- $f(x)$ is $O(g(x))$
  - $k = 1$ and $C = 4$ (witnesses)
  - $k = 2$ and $C = 3$
- $g(x)$ is $O(f(x))$
  - $k = 1$ and $C = 1$

Example

- Let $f(x) = 7x^2$ and $h(x) = x^3$
- $7x^2$ is $O(x^3)$
  - $k = 1$ and $C = 7$
- x^3 is not $O(7x^2)$
  - There is no C such that $x^3 \leq C(7x^2)$ for "large" x

Degree of Polynomial is What Matters

- Theorem:
  Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$.
  Then $f(x)$ is $O(x^n)$

More Examples

- Sum of first n positive integers
  $$1+2+\ldots+n \le \underbrace{n+n+\ldots+n}_{n} = n^2$$
  So $\sum_{i=1}^{n} i$ is $O(n^2)$
- It follows that the complexity of both bubble sort and insertion sort is $O(n^2)$

Still More Examples

- $f(n) = n!$ is the product of the first n positive integers
  $$1 \cdot 2 \cdot \ldots \cdot n \leq \underbrace{n \cdot n \cdot \ldots \cdot n}_{n} = n^n$$

  So $n!$ is $O(n^n)$