

# Discrete Mathematical Structures

## CS 3233 Lecture Eighteen

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# Business

- Assignment 6 (due Thursday, October 20, 2pm, at Dr. Winsborough's office) had a typo. The problems assigned are in Section 2.2, not Section 2.1
  - Section 2.2: 2, 4, 12, 14, 16, 18, 20, 22, 26, 30, 36, 42

# Big-Omega and Big-Theta

- Big-O provides an upper bound on function growth
- Big-Omega gives a lower bound
  - Def:  $f(x)$  is  $\Omega(g(x))$  if there are positive constants  $C$  and  $k$  such that  $|f(x)| \geq C|g(x)|$  whenever  $x > k$
- Big-Theta gives both
  - Def:  $f(x)$  is  $\Theta(g(x))$  if  $f(x)$  is  $O(g(x))$  and  $f(x)$  is  $\Omega(g(x))$
  - In this case we say  $f(x)$  is *of order*  $g(x)$

# Example

- So  $\sum_{i=1}^n i$  is  $\Theta(n^2)$ 
  - We already shown it is  $O(n^2)$ , so we just have to show it is  $\Omega(n^2)$
  - Summing only the terms greater than or equal to  $\lceil n/2 \rceil$ , we have  $n - \lceil n/2 \rceil + 1$  such terms
  - So  $1+2+\dots+n \geq (n - \lceil n/2 \rceil + 1) \lceil n/2 \rceil$   
 $\geq (n/2)(n/2) = n^2/4$

# Computational Complexity

- Time Complexity
- Space Complexity
  - Discussed more in course on data structures

# Linear Search

```
proc linear search(x:int; a1,a2,...,an: distinct ints)
```

```
  i := 1
```

```
  while (i ≤ n and x ≠ ai)
```

```
    i := i + 1
```

```
  if i ≤ n then location := i else location := 0
```

- How many comparisons are performed?

# Binary Search

**proc** binary search(x: int; a<sub>1</sub>, a<sub>2</sub>, ..., a<sub>n</sub>: increasing ints)

i:=1 {left end of search interval}

j:=n {right end of search interval}

**while** i<j **begin**

    m:=  $\lfloor (i+j)/2 \rfloor$

    if x > a<sub>m</sub> then i:=m+1 else j:= m

**end**

if x=a<sub>i</sub> then location := i else location := 0

- How many comparisons are performed?