Discrete Mathematical Structures
CS 3233 Lecture 22

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Business

- Please ask in lecture on Fridays about particularly hard homework problems that were due the previous Thursday

Problems versus Algorithms

- The complexity of a problem is characterized by the complexity of the best algorithm for solving it
- It is always possible to give a sub-optimal algorithm
  - In Chapter 3, we will see that the sorting problem is actually n log n, even though the algorithms we have discussed so far are quadratic (n^2)

Theory versus Practice

- Some algorithms that are intractable in the worst case usually execute much faster than algorithms that are always tractable
- If the intractable cases are rare enough, such algorithms may be preferable
- Thus, while Big-O analysis is helpful, it cannot be considered 100% conclusive in all cases about the relative merits of different algorithms

Undecidable Problems

- There are problems that cannot be solved by any algorithm
  - Such problems are called *undecidable*
  - The book calls them *unsolvable*
- Alan Turing first proved that the *halting problem* is undecidable
  - Given an algorithm and an input, determine whether the algorithm halts when executed on the input

Problem Classes: P versus NP

- Problems in P (polynomial time) have polynomial-time algorithms
- Problems in NP (nondeterministic polynomial time) have no known polynomial-time algorithms, but for which a solution can be checked in polynomial time
  - While it remains open whether P = NP, most computer scientists consider this unlikely

NP-Complete Problems

- There is a sub-class of NP called NP-complete for which, if any such problem can be solved in polynomial time, they all can
  - Satisfiability of propositional formulas is an NP-complete problem
    - Given a truth assignment that makes the formula true, we can verify this in polynomial time
    - There is no known algorithm to find such assignments in polynomial time
    - If one were discovered, it would prove P = NP